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VASIP 2: SOFTWARE UPDATE FOR VISUALIZING PROPAGATION OF MEASUREMENT AND PROCESS ERRORS IN VASI BASED SITE MODELS

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Abstract. This note follows Cieszewski and Wawrykowicz (2025) on VASIP 1.0 and presents VASIP 2, version 2.0.20, for variable-age site index (VASI) curves. The height model is unchanged, and the program retains sixteen historical Alberta and Saskatchewan parameter sets. New functions include: (1) animated graphs with live reference movement; (2) nested conditional bounds at one, two, and three standard errors (SEs); and (3) paired graphs of absolute and relative deviations and bound-sum bias, which measures bound asymmetry about the central curve. Site-index and height-crossing-age readouts update live with the reference point. Adjustable graphs, file imports and exports, named projects, and reporting tools further extend the workflow. Measurement-height variation and global-coefficient variation can be viewed separately or jointly at the selected sensitivity settings. In the program, Process is represented by global-coefficient variation, which changes growth efficiency (the relative height-growth rate). This propagates the coefficient uncertainty represented by the reported SEs, including any effects of lack of fit reflected in them, without simulating stochastic residual disturbances. The program includes a complete user guide; we trace its parameter sources to the 1991 Alberta note and the 1993–1995 Saskatchewan records. The bounds are deterministic sensitivity envelopes informed by marginal coefficient SEs, with no verified probability coverage.

Keywords: Variable-age site index; VASI; Base-age invariance; BAI; Self-referencing models; Polymorphic height growth; Measurement error; Process error; Error propagation; Parameter sensitivity.

1 Introduction and Background

Variable-age site index (VASI) models describe a site curve from a measured height and its age. The reference point locates the curve, while fitted global coefficients determine its form. The effects of changes in these inputs vary with prediction age. VASIP 2 makes that propagation visible by showing how reference-height measurement bounds and coefficient perturbations change the curve and its derived readouts (Figure 1).

The height model developed from work on lodgepole pine in Alberta (Cieszewski, 1988; Cieszewski and Bella, 1989, 1991). Its use of a measured height at any reference age follows the base-age-invariant approach of Bailey and Clutter (1974) and is consistent with the generalized algebraic difference approach (GADA) (Cieszewski, 2001; Cieszewski and Bailey, 2000). The equation used here is the VASI case of the three-parameter dynamic Eq. 21 of Cieszewski (2001). When fitted to the predictions of the five-parameter height growth model of Monserud (1984), it generates virtually identical curves. Related dynamic equations are discussed by Cieszewski (2003) and Cieszewski et al. (2006). The preceding paper gives the model background and derivations; Appendix A gives the height equation and its inverse as Excel, SAS, TSP, R, and C++ code.

VasiPred 1.0, presented under the name VASIP by Cieszewski and Wawrykowicz (2025), used this model to visualize the propagation of measurement and parameter errors. Its sensitivity calculation combined reference-height perturbations with a single grid of global coefficients at three standard errors (SEs) and clamped the resulting bounds to site-class curves. The clamping could

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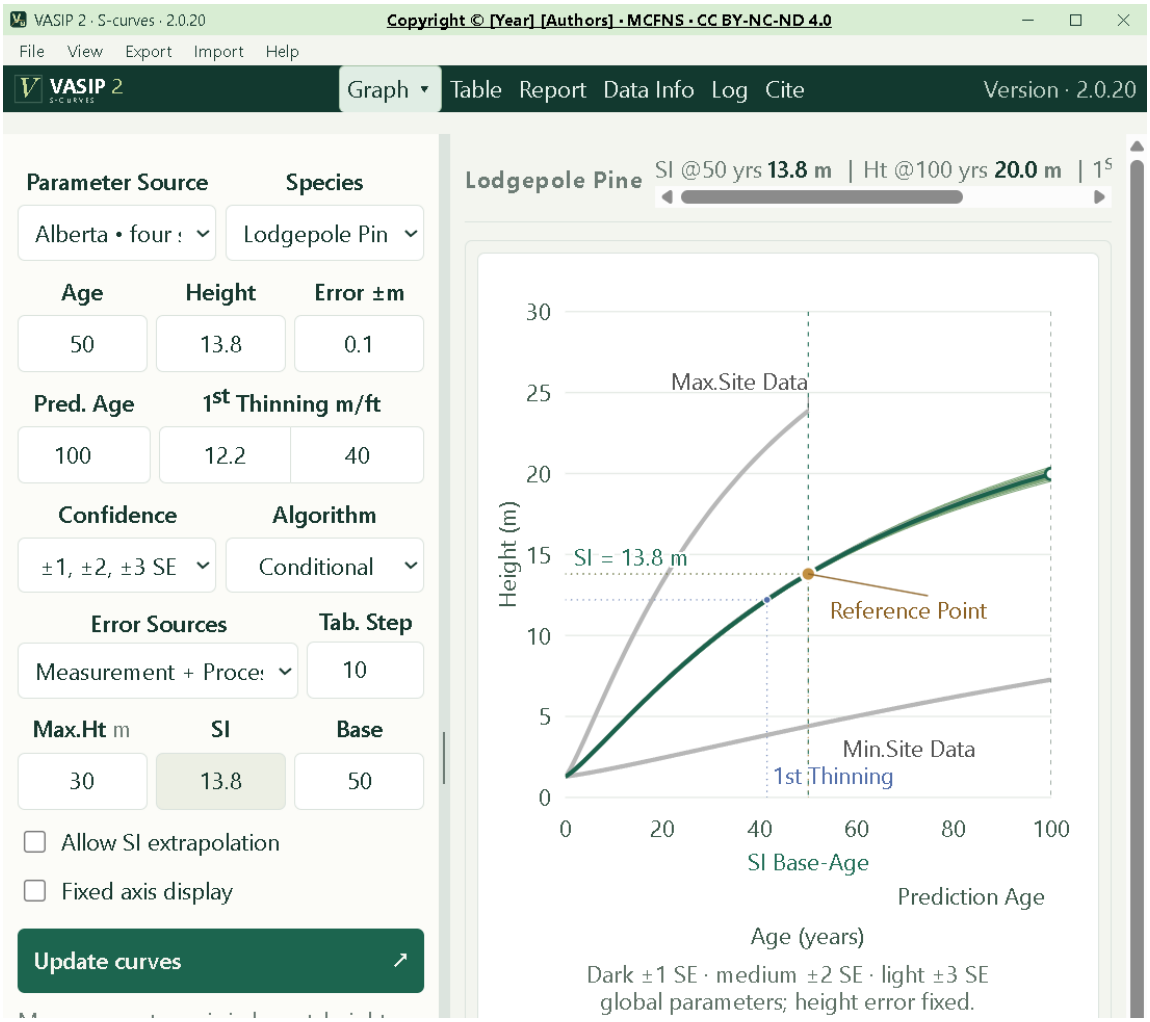

Figure 1: VASIP 2.0.20 desktop workspace with default inputs in a restored Light (green) session. The Alberta lodgepole pine reference is 13.8 m at age 50, with $e = 0.1$ m, Pred. Age 100, thinning height 12.2 m (about 40 ft), Base 50, and Max.Ht 30 m. The readouts show SI 13.8 m at age 50 and height 20.0 m at age 100. The right-edge thinning readout and lower status line are outside the captured view. This reference differs from the 12 m worked example.

conceal scenario curves beyond those guides, and a single grid did not show how the bounds changed as coefficient variation increased.

VASIP 2 retains the height model and extends this examination of error propagation. The height–age reference can be moved while the curves, site index, and height-crossing-age readouts update live. Conditional sensitivity bounds at one, two, and three SEs retain the inner grids and omit the earlier clamping. Each scenario recomputes its local site parameter, as in VasiPred 1.0. Measurement-height and global-coefficient variation can be examined separately or together, with the same fixed measurement-height anchors at every level. Legacy sensitivity mode retains the earlier single-grid construction.

Paired graphs show the signed deviations from the central curve and their bound-sum bias in meters or percent, making the asymmetry of the bounds easier to read. Development-limit displays and explicit extrapolation flags show where a curve extends beyond the stored age guides. Imports, exports, named projects, and reusable case reports support repeated comparisons. Stable evaluation, an unrounded inverse with whole-year readouts, and adaptive graph sampling improve the numerical calculations and their display.

The application is the separate S-curves (site-curves) workspace of VasiPred Desktop 3.x and runs on Windows and in a browser. It retains the sixteen historical parameter sets and fits none of them. We trace their sources to the 1991 Alberta note and the 1993–1995

Saskatchewan calibration records. The data-screening and parameter-fitting workspaces of the larger desktop program fall outside the present note, and VASIP 2 does not require TSP (Time Series Processor; Hall et al., 1992). We describe the conditional sensitivity construction, the interface, and verification of the calculations in version 2.0.20.

2 Models and Input Conventions

The notation follows Cieszewski and Wawrykowicz (2025). The symbols H_1, A_1, e, h_b, a, b, c, and q keep their meanings, with q defined at age c as in Section 2.1; for $c = 50$ this is the 2025 meaning. We write the earlier s_a, s_b, and s_c as SE(a), SE(b), and SE(c). Prediction age A_2 and height H_2 are written A and $H(A)$. The earlier clamped bounds H_L and H_U are distinguished from the present unclamped bounds L_k and U_k at level k. The 2025 combination height $H_k(A)$ is written $H_s(A)$ here, for scenario s, because k now denotes the selected level.

2.1 The reference point and height equation

A reference point consists of a total height H_1 and its breast-height age A_1. The local site parameter is found from that point and the selected global coefficients. Equations (1)–(3) give the 2025 model in the engine's stable form (Cieszewski and Wawrykowicz, 2025). The numerical engine uses the following quantities:

$$\begin{aligned} d &= bc^{-a}, \qquad h_1 = H_1 - h_b, \\ R &= \sqrt{(h_1 - d)^2 + 4bh_1A_1^{-a}}, \\ q &= \frac{h_1 - d + R}{2}, \end{aligned} \tag{1}$$

where:

- d is bc^{-a}, computed from the global coefficients;
- a is the first positive global model coefficient;
- b is the second positive global model coefficient;
- c is the third positive global model coefficient, the age at which q is defined;
- h_1 is the reference height above the base height;
- H_1 is the measured total reference height in meters;
- h_b is the base height, 1.3 m in the bundled sets;
- A_1 is the positive reference breast-height age in years;
- R is the positive square root defined above; and
- q is the positive local site parameter implied by the reference point.

When $h_1 - d$ is negative, the engine computes the algebraically equivalent root $q = 2bh_1A_1^{-a}/[R - (h_1 - d)]$. This avoids subtraction of nearly equal numbers. The local parameter is calculated from the reference point. It is not another independently varied global coefficient.

The height at a positive prediction age is:

$$H(A) = h_b + \frac{q(q + d)}{q + bA^{-a}}, \tag{2}$$

where:

- $H(A)$ is predicted total height in meters;
- A is breast-height age in years;
- h_b is the base height;
- q is the local site parameter;
- d is bc^{-a};
- c is the third global coefficient, the age at which q is defined;
- a is the first global coefficient; and
- b is the second global coefficient.

Equations (1)–(2) are Eq. 21 of Cieszewski (2001) applied to heights above the base height, $Y = H - h_b$, with $\delta = a$, $\kappa = 2b$, and $\eta = bc^{-a} = d$. With this constraint, q is the height above the base at age c; code for Eq. 21 in the VASIP 2 form is given in Appendix A.

For positive ages, differentiating $Y(A) = H(A) - h_b$ in Eq. (2), with the coefficients and reference point held fixed, gives:

$$\begin{aligned} \frac{\mathrm{d}H}{\mathrm{d}A} &= \frac{\mathrm{d}Y}{\mathrm{d}A} = Y(A)\frac{abA^{-a-1}}{q + bA^{-a}}, \\ \frac{1}{Y}\frac{\mathrm{d}Y}{\mathrm{d}A} &= \frac{abA^{-a-1}}{q + bA^{-a}}, \\ \frac{1}{H}\frac{\mathrm{d}H}{\mathrm{d}A} &= \frac{Y(A)}{H(A)}\frac{abA^{-a-1}}{q + bA^{-a}}. \end{aligned}$$

The second line is the relative growth rate above the base height; the third is the relative rate of total height, termed growth efficiency here. Conditioning the curve on (A_1, H_1) through q fixes its height at A_1, while the global coefficients and the implied q govern growth efficiency along the trajectory. Process mode perturbs the global coefficients (a, b, c) and recomputes q from the same reference point (Section 3). The calculation is deterministic and does not inject or simulate residual process noise or stochastic growth disturbances.

Equation (2) is equivalent to the self-referencing height equation in Cieszewski and Wawrykowicz (2025). The engine evaluates its log-logistic ratio in a form that avoids large intermediate powers. It returns H_1 exactly when $A = A_1$ and returns h_b at $A = 0$. The local parameter q is height above the base at age c. For $c = 50$, it is site index above breast height at breast-height age 50. For New Jack Pine, $c = 1$, so q is height above the base at age 1. Its site index at age 50 is computed from Eq. (2). Table 1 gives the file SI50 guides (total heights at breast-height age 50) and development ages. Table 1 of Cieszewski and Wawrykowicz (2025) gives the coefficients and their SEs.

Table 1: File SI50 guide heights and development ages in the sixteen bundled VASIP 2 parameter sets. The bundled Alberta guides differ from the corrected 2025 total-height values without affecting the coefficients or central height equation; users who need the 2025 corrected guides can import the corrected `.DEF`/`.LIM` files described in Section 2.3. Table 1 of Cieszewski and Wawrykowicz (2025) gives the coefficients and their SEs.

Species (code)	SI50 (m)			Development age (years)		
	High	Middle	Low	High	Middle	Low
Alberta (ALTAVASI)						
Lodgepole pine (lP)	23.88	13.83	4.4	50	100	260
White spruce (wS)	24.45	13.83	5.01	50	105	250
Black spruce (bS)	15.87	9	4	50	95	190
Trembling aspen (tA)	25	17.5	8.3	50	70	140
Saskatchewan (VASIP_12)						
Lodgepole pine (lP)	19.51	13.97	4.73	100	105	130
Jack pine (jP)	18.4	13.84	8.72	90	130	60
Black spruce (bS)	18.24	11.07	5.21	90	140	268
White spruce (wS)	21.96	14.77	6.2	100	275	80
Manitoba maple (mM)	18.02	14.02	10.51	50	70	70
Trembling aspen (tA)	23.21	15.74	7.6	70	110	80
White elm (wE)	18.27	13.32	8.69	80	120	200
Balsam fir (bF)	20.12	13.3	6.29	80	90	120
Balsam poplar (bP)	23.28	16.05	7.01	60	90	194
White birch (wB)	20.29	13.32	7.61	50	120	245
Tamarack larch (tL)	16.22	9.74	5.24	50	120	140
New Jack Pine (nP)	16.22	9.74	5.24	90	130	60

Note: The program treats these heights as total heights, and ages are breast-height years. High, Middle, and Low identify the stored SI50 guide classes and their associated age limits. These file values do not all reproduce the corrected 2025 table or the original data summaries; see Section 2.3 and Appendix B. The gray site-data guides are its Max, Avg, and Min curves. Coefficients a, b, and their SEs are those of the 2025 table at the program's full file precision. All base heights are 1.3 m, SE(c) = 0, and $c = 50$ except New Jack Pine, with $c = 1$. Its coefficients, absent from that table, are $a = 1.22202$, SE(a)= 0.014599, $b = 24.031$, and SE(b)= 0.637332. ALTAVASI retains the uncorrected Alberta guides, including 13.83 m for the white spruce middle guide. New Jack Pine is absent from the 2025 paper's eleven-species Saskatchewan set and supplement. Its LIM row repeats tamarack larch SI50 values and jack pine ages. Its coefficients and SEs match the March 1995 archive run `PJP22`, described in Section 2.3. All sets retain their legacy exploratory status.

The age at a requested total height is obtained by inversion:

$$A_T = \left[\frac{bh_T}{q(q+d-h_T)}\right]^{1/a}, \tag{3}$$

where:

- A_T is the breast-height age at the requested height;
- h_T is the requested height above the base height, $H_T - h_b$;
- H_T is the requested total height in meters;
- q is the local site parameter;
- d is bc^{-a};
- c is the third global coefficient, the age at which q is defined;
- a is the first global coefficient; and
- b is the second global coefficient.

These equations are given as Excel, SAS, TSP, R, and C++ code in Appendix A. Equation (3) applies when $0 < h_T < q + d$. At $H_T = h_b$ the engine returns zero. A height below the base or at or above the asymptote is reported as unreachable. The stable root and exact inverse are the implemented forms discussed in Section 5. The interface calls this result "1st Thinning." It is the crossing age of a user-selected height. It does not estimate a treatment response or determine an optimal thinning age.

2.2 Breast height and site index

The breast-height convention is the same as in the preceding paper. Ages are the years since the tree reached 1.3 m. Heights entered and reported are total heights measured from the ground. The engine subtracts the base height before calculating the local parameter and adds it to the predicted height. A total tree age is not entered in place of breast-height age and is not converted automatically. The program does not estimate the years needed to reach breast height.

The reference height and its lower measurement-error anchor must exceed the base height. Reference age is positive and at most 10,000 breast-height years. On commitment, Age, Pred. Age, and Base round to whole years. Height and Error ±m round to 0.1 m. Pred. Age accepts zero through 10,000 years and sets the plotted range, rounded up to an age tick. Base accepts 1–1,000 years. 1st Thinning m/ft has linked inputs with one decimal in each unit. One foot is 0.3048 m, so 12.2 m is about 40 ft. These committed values are the calcula-

tion inputs and are saved in projects. Model coefficients and calculated exports keep full precision.

The three readouts beside the species name show SI at Base and height at Pred. Age to 0.1 m, and the 1st Thinning @ readout in whole years. Calculations use the unrounded inverse. The thinning point's hover label shows the crossing age to 0.1 year. The CSV has no crossing-age column. The Table tab shows Age, Height, Lower, and Upper at the whole-year Tab. Step interval, which defaults to 10 years. It also includes the reference, Base, and prediction ages. Heights in that table are shown to 0.1 m.

The editable **Base** control selects the reporting age for site index. It changes $H(\text{Base})$ without changing the curve. The site-domain check and development-age limit still use height at age 50. Changing the reporting age does not redefine the bundled limits.

2.3 Bundled parameter collections

Table 1 lists the sixteen shipped parameter sets, their file SI50 guides and development ages. ALTAVASI contains four Alberta parameter sets. VASIP_12 contains twelve parameter sets for eleven Saskatchewan species, including separate Jack Pine and New Jack Pine sets. All base heights are 1.3 m. The parameterizations fix c at 50 for fifteen sets and at 1 for New Jack Pine; $\text{SE}(c) = 0$ therefore disables its variation. The software labels these legacy parameters as exploratory. We describe the coefficient sources below. The files hold only the coefficients and their marginal SEs. No coefficient covariance matrix is present in the files or the fitting archive.

The Alberta sets come from Forest Management Note 51 (Cieszewski and Bella, 1991), which fitted dominant and codominant tree stem analyses by nonlinear least squares. The Saskatchewan records include the SHAZAM draft of Cieszewski et al. (1993) and the later report and TSP fitting archive (Cieszewski, 1995; Hall et al., 1992). Table 2 summarizes the source fits. Appendix B records the screening, parameter conversions, fitting sequence, and discrepancies between the source records and bundled files.

Ten of the eleven Saskatchewan coefficient pairs and their SEs match the final April 1995 TSP runs. The bundled aspen values come from a run absent from the archive. The final white spruce and balsam fir runs, and the archived aspen run, did not report convergence within 40 iterations. New Jack Pine matches the March 1995 run `PJP22`, a $c = 1$ parameterization using the same 116 jack pine trees. It is absent from the 2025 paper's eleven-species Saskatchewan set and supplement.

The stored development ages are coarse guides. In Alberta they derive from the minimum, rounded mean, and maximum ages of the fitting trees; several Saskatchewan limits exceed the oldest tree in the corresponding data. They do not define verified prediction domains. Some SI50 guides also differ from the source statistics (Appendix B).

The Alberta guides remain as stored in version 2.0.20. Correcting them changes the guide curves, range checks, and interpolated development ages. With unchanged coefficients and the same height–age reference, it leaves the central height trajectory unchanged. The corrections include the breast-height convention and differences in stored values; they are not uniformly an addition of 1.3 m (Section 6).

The corrected pair is in the folder `ParameterSets/Alberta` of the 2025 paper's `VasiPred_1.0.zip` supplement, as `VASIPRED.DEF` and `VASIPRED.LIM`. Select Import > Matching .DEF + .LIM in VASIP 2 and open both files together. The import command of version 2.0.20 accepts all four models from this pair. It preserves the corrected total-height guides, namely lodgepole pine 25.18/15.13/5.70 m, white spruce 25.75/15.52/6.31 m, black spruce 17.17/10.29/5.28 m, and trembling aspen 26.23/18.77/9.58 m. The supplement is on the journal page of the preceding paper, article 17.4 (Cieszewski and Wawrykowicz, 2025).

3 Conditional Error Propagation

3.1 Scenario sets

The conditional bounds use fixed measurement-height anchors and separate grids of global coefficients (Process mode perturbs global coefficients only and does not generate residual growth disturbances; Section 3.3). The entered error e is a nonnegative height bound in meters. It is the same at every level. The reference age and base height stay fixed. The sets for combined measurement and coefficient variation are:

$$\begin{aligned} \mathcal{H}_e &= \{H_1 - e, H_1, H_1 + e\}, \\ \mathcal{T}_{j,r} &= \{\theta_j - r\,\text{SE}(\theta_j), \theta_j, \theta_j + r\,\text{SE}(\theta_j)\}, \\ \mathcal{T}_r &= \mathcal{T}_{a,r} \times \mathcal{T}_{b,r} \times \mathcal{T}_{c,r}, \\ \mathcal{C}_k &= \bigcup_{r=0}^{k} (\mathcal{H}_e \times \mathcal{T}_r), \end{aligned} \tag{4}$$

where:

- $\mathcal{H}_e$ is the set of fixed total-height anchors;
- H_1 is the measured total reference height;
- e is the fixed measurement-height bound;
- θ_j is the central value of coefficient j, with $j = a, b, c$;
- $\text{SE}(\theta_j)$ is the reported standard error of coefficient j;
- $\mathcal{T}_{j,r}$ is its set of perturbed values at grid level r;
- $\mathcal{T}_r$ is the three-coefficient grid at level r;
- $\mathcal{C}_k$ is the combined scenario set through level k;

Table 2: Calibration summaries behind the bundled parameter sets: Forest Management Note 51 (Alberta) and the 1995 TSP fitting archive (Saskatchewan).

Species (code)	Trees	Observations	SE of residuals (m)	R^2	ρ
Alberta (ALTAVASI)					
Lodgepole pine (lP)	1163	–	0.84	0.970	–
White spruce (wS)	698	–	1.21	0.965	–
Black spruce (bS)	282	–	0.66	0.969	–
Trembling aspen (tA)	276	–	0.69	0.977	–
Saskatchewan (VASIP_12)					
Lodgepole pine (lP)	71	627	0.340877	0.996342	1.19092
Jack pine (jP)	116	904	0.390823	0.993927	1.20078
Black spruce (bS)	55	527	0.276667	0.997414	1.21568
White spruce (wS)[b]	132	1188	0.501652	0.995380	1.20531
Manitoba maple (mM)	9	59	0.342220	0.995136	0.804903
Trembling aspen (tA)[a,b]	31	226	0.573171	0.991461	0.805381
White elm (wE)	17	187	0.376756	0.995557	1.12756
Balsam fir (bF)[b]	34	254	0.363616	0.996965	1.30881
Balsam poplar (bP)	37	326	0.485170	0.994461	1.21006
White birch (wB)	37	327	0.408247	0.994090	1.19581
Tamarack larch (tL)	8	100	0.399746	0.990665	1.29712
New Jack Pine (nP)	116	904	0.367022	0.994666	1.19521

Note: Alberta values are from Tables 2 and 4 of Cieszewski and Bella (1991); the note's SE is the reported SE of height prediction, and observation counts and ρ are not given. Saskatchewan values are from the final simultaneous TSP fits, including TSP's R^2; New Jack Pine is run `PJP22`. The fitted ρ is the coefficient of the lagged decadal residual of the same tree. It is not stored in the parameter files. [a]The archived aspen run has 31 trees. The file's aspen coefficients come from a run that is not in the archive. [b]TSP did not report convergence within the 40-iteration limit. These archived-run statistics do not establish convergence or biological predictive performance.

- r is an integer grid level from zero through k; and
- k is the selected level, zero, one, two, or three.

The central global-coefficient vector is $\boldsymbol{\theta} = (a, b, c)$. Each scenario $s = (H^*, \boldsymbol{\vartheta}) \in \mathcal{C}_k$ recomputes q from its total-height anchor $H^* \in \mathcal{H}_e$ and scenario coefficient vector $\boldsymbol{\vartheta} \in \mathcal{T}_r$ through Eq. (1). Its height curve uses Eq. (2). The local parameter is never independently perturbed. Every valid scenario passes through its own anchor, so $H_s(A_1) = H^*$. Changes in $\boldsymbol{\vartheta}$ and the recomputed q can alter growth efficiency (Section 2.1) and thus the scenario trajectories away from their anchors.

The program pools separate grids. It never combines $a + 1\,\mathrm{SE}(a)$ with $b - 3\,\mathrm{SE}(b)$ in one scenario. Repeated values are included once. In the shipped sets, a and b have positive SEs and $\mathrm{SE}(c) = 0$. With $e > 0$, combined levels 1, 2, and 3 contain 27, 51, and 75 unique scenarios. The corresponding process-only counts are 9, 17, and 25. Measurement-only calculations have three anchors.

VasiPred 1.0 evaluated 81 combinations for the same three height anchors at one three-SE level (Cieszewski and Wawrykowicz, 2025). It repeated the zero-SE c values. Removing these repetitions gives 27 unique combinations in a single nonzero grid. VASIP 2 conditional level 1 has 27 scenarios. Levels 2 and 3 also retain the inner grids. The count comparison does not equate one-SE bounds with three-SE bounds.

3.2 Bounds and nesting

The displayed bounds are the smallest and largest scenario heights:

$$L_k(A) = \min_{s \in \mathcal{C}_k} H_s(A), \qquad U_k(A) = \max_{s \in \mathcal{C}_k} H_s(A), \tag{5}$$

where:

- $L_k(A)$ is the lower bound at age A and level k;
- $U_k(A)$ is the upper bound at that age and level;
- $\mathcal{C}_k$ is the combined set defined in Eq. (4), or its measurement-only subset (central coefficients) or process-only subset (single anchor H_1); and
- $H_s(A)$ is Eq. (2) localized to scenario s.

For process only, the same construction uses the single anchor H_1. For measurement only, it uses the central coefficients and $\mathcal{H}_e$. Retaining the inner grids makes the zones nested. A nonlinear extremum can occur at an inner combination. Invalid requested permutations produce an error rather than being silently omitted. A valid lower level can still be used.

At the reference age, every process-only scenario returns H_1. Every combined level spans the same $H_1 - e$ to $H_1 + e$. The measurement-only band has these same bounds at every level. The entered error is not a standard deviation and is not multiplied by one, two, or three.

The horizontal error view subtracts the central height at the same age from each bound and sums the two signed deviations:

$$D_k^-(A) = L_k(A) - H(A), \quad D_k^+(A) = U_k(A) - H(A),$$
$$D_{k,\mathrm{rel}}^-(A) = 100\,\frac{D_k^-(A)}{H(A)}, \quad D_{k,\mathrm{rel}}^+(A) = 100\,\frac{D_k^+(A)}{H(A)},$$
$$B_k(A) = D_k^-(A) + D_k^+(A), \quad B_{k,\mathrm{rel}}(A) = 100\,\frac{B_k(A)}{H(A)}, \tag{6}$$

where:

- $D_k^-(A)$ is the signed lower deviation in meters;
- $D_k^+(A)$ is the signed upper deviation in meters;
- $L_k(A)$ is the lower conditional bound;
- $U_k(A)$ is the upper conditional bound;
- $H(A)$ is the central height at the same age, nonzero for the relative values;
- $D_{k,\mathrm{rel}}^-(A)$ is the lower deviation in percent of the central height;
- $D_{k,\mathrm{rel}}^+(A)$ is the upper deviation in percent of the central height;
- $B_k(A)$ is the bound-sum bias in meters;
- $B_{k,\mathrm{rel}}(A)$ is the bound-sum bias in percent;
- A is breast-height age in years; and
- k is the selected coefficient level.

Relative values are omitted where $H(A) = 0$, which can occur only for an imported set with a zero base height. This bias measures bound asymmetry; $B_k(A)/2$ is the displacement of the bounds' midpoint from the central height. It is not an estimate of expected statistical bias. At A_1, combined and measurement-only deviations are $-e$ and $+e$. Their relative values are $\pm 100e/H_1$. Process-only deviations are zero there. The bias is zero at A_1 for every source.

Figure 2 shows the program's absolute and relative double graphs for the default lodgepole pine reference. The two panels in each graph share one age scale in a square frame. At age 50, every combined level spans -0.10 to $+0.10$ m, or -0.72 to $+0.72$%, with zero bias. At age 100, $H = 19.971$ m. Levels 1, 2, and 3 have deviations of $-0.227/+0.227$, $-0.337/+0.335$, and $-0.447/+0.441$ m. The level-3 relative deviations are $-2.24/+2.21$%. The biases at that age are $+0.0002$, -0.0017, and -0.0060 m. Over ages 0–100, the largest absolute biases in the sampled output are 0.0022 m at age 21.25 for level 1, 0.0068 m at age 21 for level 2, and 0.0139 m at age 21 for level 3. The largest sampled relative bias is 0.22% near age 15.75 for level 3. These peak ages depend on the sampling grid and are approximate. At age 10, $H = 4.10$ m and the level-3 deviations are only $-0.073/+0.076$ m, but $-1.78/+1.86$%. The percentages can be large at young ages even when the meter deviations are small. The bias curves bend between ages 13 and 16. There the scenarios that give the lower and upper bounds change. At age 100, process-only level 3 gives $-0.331/+0.320$ m. Measurement only gives $-0.119/+0.119$ m.

Figure 3 shows conditional bounds for Saskatchewan tamarack larch at 15.5 m and 66 years, with $e = 0.1$ m. All levels span 15.4–15.6 m at that reference age. At Base 50, SI is 13.6 m. The level-1 band is 13.13–14.19 m and the level-3 band is 12.56–14.98 m. The readouts give 19.2 m at age 140 and a thinning age of 41 years.

3.3 Separate sources

Table 3 summarizes the four choices in the **Error Sources** control. The program uses Process to mean perturbations of the global coefficients based on their SEs. It does not sample a residual growth error.

Setting $e = 0$ removes measurement variation. It does not remove coefficient variation. Measurement only with $e = 0$ gives no visible area. No band leaves the central curve visible. The source choices allow the two contributions to be examined separately.

3.4 Legacy mode

The **Algorithm** control has **Conditional** and **Legacy sensitivity**. The conditional option gives the nested bounds described above. The legacy option uses one selected coefficient multiplier and the same fixed measurement-height bound. It shows one band and does not retain all inner grids. Unlike VasiPred 1.0, VASIP 2 retains and counts scenarios outside the supplied SI50 range. It does not clamp the bounds to the extreme site guides. Bounds can therefore differ from the older program near a site-class limit. Figure 4 compares the program's graphs in conditional and legacy modes. It uses the preceding paper's lodgepole pine reference and error bound, with Pred. Age set to 150 to show the longer range. The three conditional zones are drawn from light to dark. They meet the same fixed measurement bounds at age 50. The outer bounds in this example coincide with the selected three-SE legacy bounds.

Table 3: Error sources and the width at the reference age.

Source	What varies	Width at A_1
Measurement + Process	Fixed height anchors and global coefficients	$2e$
Process only	Global coefficients, with anchor H_1	Zero
Measurement only	Fixed height anchors, with central coefficients	$2e$
No band	Central curve only	No shading

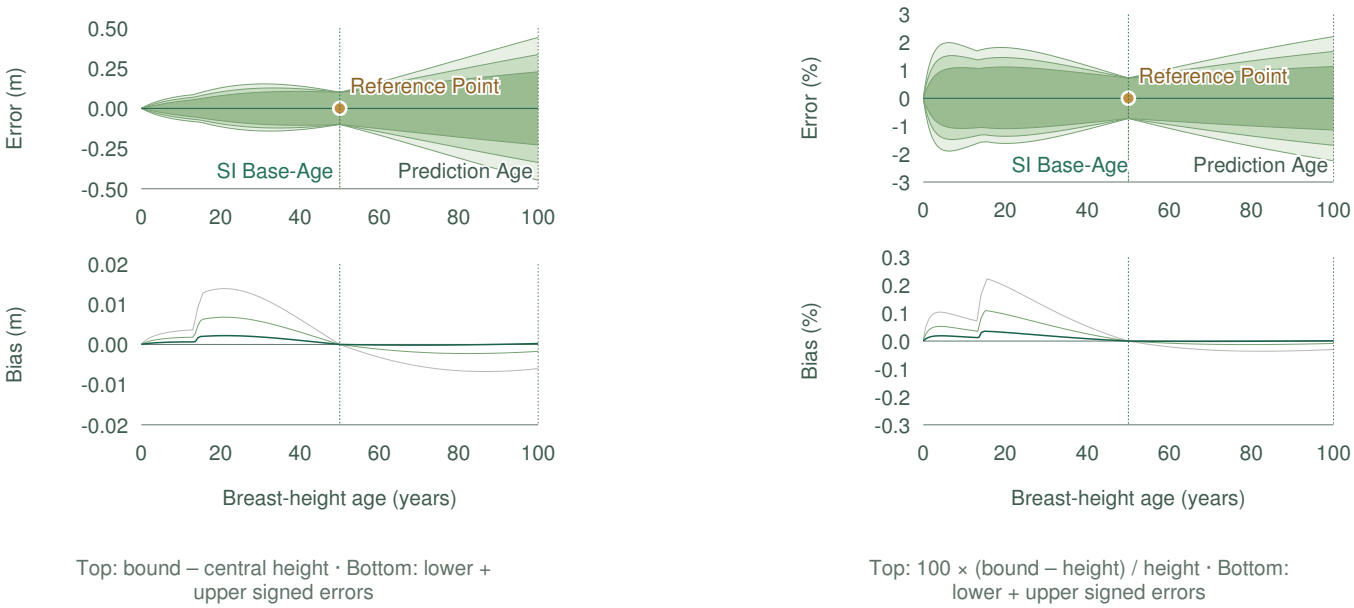

Figure 2: VASIP 2.0.20 Horizontal errors + bias graphs in the explained design and Light (green) scheme. Left: absolute units (m). Right: relative units (%). The upper panels show conditional deviations from the central height. The lower panels show their signed sum. The bias is a bound-asymmetry measure. Shading from dark to light marks levels 1, 2, and 3. In the lower panels, the dark green, light green, and gray lines are the level-1, 2, and 3 biases. The reference is Alberta lodgepole pine, $H_1 = 13.8$ m at $A_1 = 50$ years, with $e = 0.1$ m, combined levels 1–3, and Pred. Age 100. All values were computed by the shipped engine.

3.5 Interpretation

The control is named Confidence, but the levels specify coefficient perturbations. We keep that control name and the term bound-sum bias so that the text matches the program; in this paper both are sensitivity terms. These are sensitivity bounds with no verified probability coverage. Levels 1, 2, and 3 do not mean 68%, 95%, and 99.7% probability coverage. The grids do not supply a joint coefficient distribution, covariance, residual process error, or age-measurement error. Nor do they find all extrema in a continuous coefficient region. The same sensitivity interpretation applies to the older envelope (Cieszewski and Wawrykowicz, 2025).

4 The Program

4.1 Workspace and workflow

Figure 1 shows the VASIP 2.0.20 window with the default inputs. The banner gives the version and six tabs: Graph, Table, Report, Data Info, Log, and Cite. The species name and three readouts share the heading above the graph. Graph includes the design and units menu. The released title row, Cite tab, and exports retain article-citation placeholders; Section 7 gives the citation instructions.

The left panel has **Parameter Source** next to **Species**. The reference inputs are **Age**, **Height**, and **Error ±m**. **Pred. Age** shares a row with linked **1st Thinning m/ft**. **Confidence** (which selects sensitivity levels: No Process variation, level 1, levels 1 and 2, or levels 1, 2, and 3) and **Algorithm** (Conditional or Legacy sensitivity) share the next row. **Error Sources** shares a row with **Tab. Step**, followed by **Max.Ht m**, read-only **SI**, and **Base**. Below these sit Allow SI extrapolation, Fixed axis display, Update curves, and collapsible Import & save and Model provenance sections.

The program commits an edited value when the user leaves its field or selects **Update curves**. Ages commit to whole years and heights to 0.1 m. Base changes the reporting age of SI. Max.Ht sets the upper height limit. With **Fixed axis display** selected, the age and height ranges stay unchanged when the inputs change. These controls affect reporting or the graph scale. Allow SI extrapolation permits calculation when the reference's implied SI50 is outside the supplied range. Without this option, the program rejects that reference. With the option on, the range warning remains. The thinning readout is a height-crossing age, rather than a treatment response or management prescription.

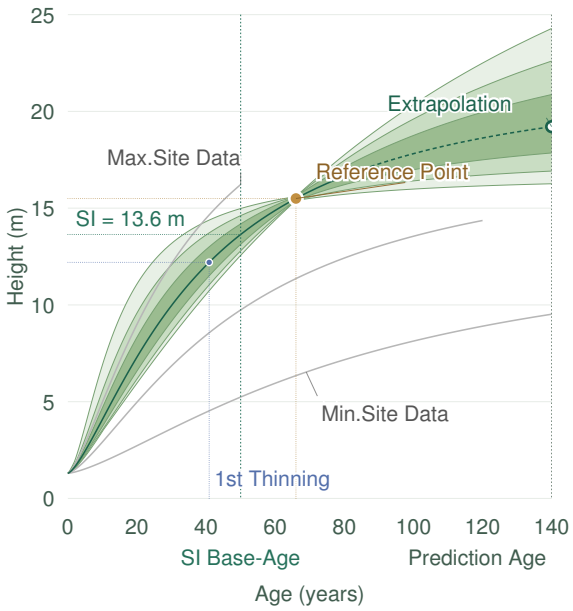

Figure 3: VASIP 2.0.20 height graph for Saskatchewan tamarack larch in the Light (green) scheme. The committed reference is $H_1 = 15.5$ m at $A_1 = 66$ years, with $e = 0.1$ m, Pred. Age 140, thinning height 12.2 m, Max.Ht 25 m, and conditional levels 1–3. The zones meet the fixed measurement bounds at the reference. At Base 50 the level-3 band is 12.56–14.98 m. The curve is dashed beyond its interpolated development limit of 77.9 years. The unlabeled gray curve is the middle SI50 guide.

4.2 Reading the graph

The three gray site-data guides show the high, middle, and low SI50 classes. Each stops at its reported maximum development age or at the plot end. LIM supplies no minimum development ages. The selected curve is solid up to a limit interpolated between its adjacent SI50 classes and dashed beyond it. Exact class values use that class's age. The interpolation permits nonmonotone class ages.

Figure 5 shows Saskatchewan trembling aspen at the committed reference of 15.7 m and age 50. Its file middle SI50 guide remains 15.74 m. The guides' high, middle, and low development ages are 70, 110, and 80 years. The selected curve becomes dashed after its interpolated limit of 109.9 years. The level-3 band at age 50 is 15.4–16.0 m. The other guides stop at their own limits. A reference of 19.5 m at age 50 gives an interpolated limit of 89.9 years, between the high and middle limits.

Static vertical guides mark the SI base age and prediction age. Dotted coordinate lines mark the reference, SI, and smaller blue thinning point. Imported observations appear as an overlay. The age and height axes have equal physical lengths, with labels and caption margins outside the square. Years and meters retain different numerical scales. Equal axis lengths do not make their units equal. Pred. Age sets the plotted age range, rounded up to an age tick. For example, 150 years gives a graph ending at 160 years. A target-height crossing beyond the plot can still have a reported age.

A reference outside the supplied SI50 range needs the extrapolation override. Dragging outside that range enables the override and displays its warning. Outside the range, the development-age calculation holds the nearest endpoint age. Scenario curves outside the SI50 range remain in the bounds and are counted. The warning identifies extrapolation without establishing its suitability for a stand.

The banner Graph menu offers Height trajectories and Horizontal errors + bias, each in clean or explained form. Clean removes in-plot annotations, legends, and explanatory text. Explained keeps labels and adds text below the plot. Absolute errors (m) and Relative errors (%) select the units and switch to the horizontal display. Selecting a horizontal design activates conditional bounds. Under Legacy sensitivity, the horizontal views in a case report are labeled a Conditional comparison. Error and bias scales fit their selected bounds independently of Max.Ht.

4.3 Live reference movement

Dragging the gold reference point or its label in a height view changes its age and height. In either error view, the drag changes age only and holds height fixed. The curve, bounds, SI, prediction, and threshold age update during the drag. The table refreshes on release. The global coefficients, measurement bound, Base guide, and Prediction Age guide stay fixed. A focused reference handle can also be moved with arrow keys. Left and right change age by one year. Up and down change height by 0.1 m. Shift multiplies the step by ten. Escape cancels a drag. The axes stay fixed while dragging and normally rescale when it ends.

Figure 6 shows the default lodgepole pine curve before and after moving its reference. At 50 years and 13.8 m, the readouts are SI 13.8 m, height 20.0 m at 100 years, and thinning at 42 years for the 12.2 m threshold. Its interpolated development limit is 100.5 years. Moving the reference to 80 years and 24 m changes the readouts to 19.1 m, 26.1 m, and 25 years. The moved reference is beyond its 73.6-year development limit, on the dashed segment labeled *Extrapolation*. The coefficients and fixed $e = 0.1$ m stay unchanged.

A recorded desktop drag used 18 frames. SI changed at every frame, with median response 9.6 ms and largest

response 18.7 ms. All checks were true, including the site-line color check and the overall ready flag. These times describe that run on the first author's x64 Windows 11 computer.

4.4 Imports, exports, and projects

Matching DEF and LIM files are imported together. DEF supplies coefficients, SEs, and base height. LIM supplies names, SI heights, and maximum ages. The parser matches species codes rather than row positions. It rejects missing, duplicate, or invalid entries. Binary PAR files and legacy prediction-report DAT files are not direct inputs. Observation CSV needs exactly the header `age,height`, in breast-height years and total meters. Missing or nonfinite cells are rejected. Files are limited to 5 MB each and observations to 10,000 records. Loading observations does not fit or change coefficients.

Right-clicking the graph offers Copy graph, Print graph, and Save As. Copy graph makes a 1600-by-1600 bitmap of the plotting area inside the axes. Save As offers SVG, PNG, JPEG, and PDF. With Dark selected, saving offers Dark as displayed or Light as a paper version. This choice leaves the screen appearance unchanged. SVG preserves the vector graph and labels. CSV records full-precision predictions and bounds, inputs, coefficients, SEs, provenance, and extrapolation flags. The fixed error is `height_error_bound_m`, while the older measurement-error column stays blank. Unselected higher-level bounds and guides beyond their development ages are blank. Print/PDF opens the print dialog for a static copy. Invalid inputs clear old results and disable exports until corrected.

Named JSON projects keep the model, inputs, graph settings, and observations. New files use `vasip2-project`, version 1. The loader also accepts `vasipred-project`, version 1, curve projects. Older propagated settings migrate to conditional mode. Legacy sensitivity settings retain their mode. The desktop keeps its own WebView2 profile and last-session file under `%LOCALAPPDATA%\VASIP2`. On first use it can transfer a saved VasiPred S-curves setup. An edited VASIP 2 session is retained. Each user's session stays on that user's computer. Sessions and named projects keep the graph design, explanation choice, and units. Pane borders can be resized, and the desktop restores their sizes.

The standalone browser edition has curve import and export controls and uses named projects for saving and

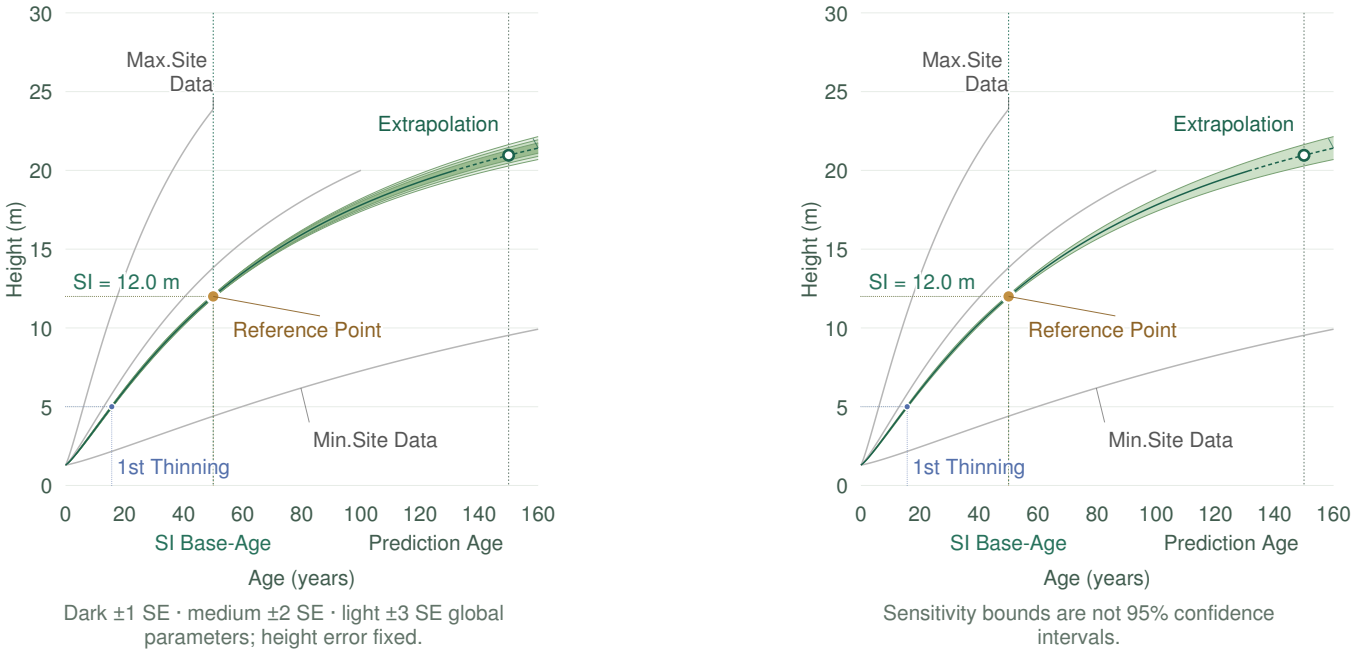

Figure 4: VASIP 2.0.20 graphs in the Light (green) scheme for Alberta lodgepole pine, $H_1 = 12$ m, $A_1 = 50$ years, $e = 0.1$ m, prediction age 150, thinning height 5 m, and Max.Ht override 30 m. Left: conditional levels 1, 2, and 3. Right: legacy sensitivity bounds at multiplier 3. The plot ends at the tick-aligned age 160. The readouts are SI 12.0 m, height 21.0 m at age 150, and thinning at 16 years. The development limit is 131.0 years. The gray guides use the unchanged bundled ALTAVASI limits. The unlabeled gray curve is the middle SI50 guide. The outer bounds of the two panels coincide here because both algorithms give the same outer values for this example (Section 5.4). Conditional and legacy modes differ where inner-grid extrema fall inside the outer combinations. Comparisons with VasiPred 1.0 also differ where its clamping to site-class curves applies.

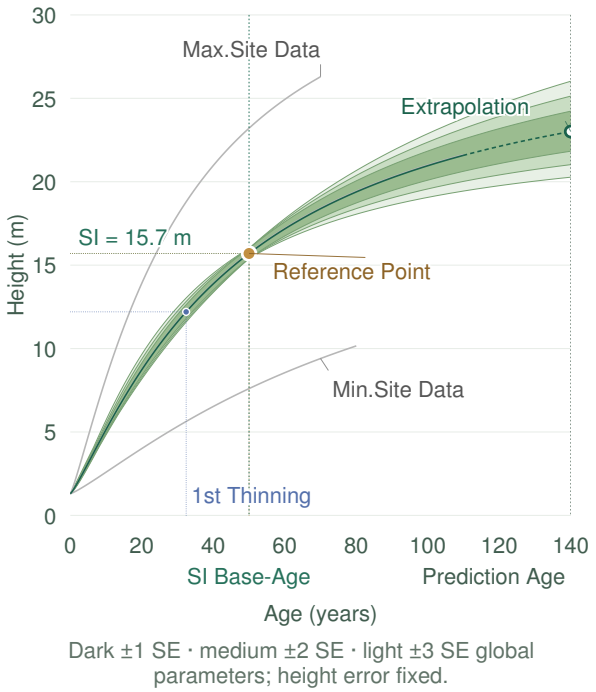

Figure 5: VASIP 2.0.20 output for Saskatchewan trembling aspen in the Light (green) scheme. The committed reference is $H_1 = 15.7$ m at age 50, with $e = 0.3$ m, Pred. Age 140, thinning height 12.2 m, and Max.Ht 30 m. Guide limits are 70, 110, and 80 years. The selected curve becomes dashed after 109.9 years. The readouts show SI 15.7 m, height 23.0 m at 140 years, and thinning at 32 years. The middle SI50 guide nearly coincides with the selected curve.

reopening. Desktop automatic restart state does not apply to it. Both editions calculate and import local curve files without an account, server, or network connection. Their packaged HTML files are byte-identical and contain the same JavaScript computational and graphing code. Section 5.5 distinguishes shared numerical checks from host-specific behavior. Section 7 gives the desktop requirements and installation steps.

View > Graphic interface offers Light (green), Dark, Light (blue), and High contrast (grayscale). Dark is the default. Seven fonts are available, with 12 pt interface text. Exports retain the selected scheme. Every scheme keeps three progressively lighter zones. The figures here use Light (green).

The Table tab uses Tab. Step for the age interval. Report follows the first author's graduate report template: Results contains height, absolute-error, and relative-error graph snapshots, current-case results, and the error table; the prediction table appears in Discussion. User writing persists as the case changes. Copy Report includes the writing, graphs, tables, and text; Save Report writes self-contained HTML without editor controls or guidance. Data Info lists coefficients, SEs, guides, ages, and provenance. Log retains session activity and desktop startup records and offers Save Log. Cite provides citation and copyright text.

4.5 Help

The complete user guide opens from the desktop Help menu or F1. It covers inputs, graph designs and units, sensitivity bounds, file operations, reports, appearance, scientific limits, and troubleshooting. The menus provide project saving and loading, graph and table exports, printing, imports, and display controls. Reset pane sizes restores the default layout. The browser edition uses the separate `www\app\guide.html` file in the installation folder. Section 2.3 gives the corrected-Alberta import procedure.

5 Numerical Methods and Verification

5.1 Changes relative to VasiPred 1.0

Equations (1)–(3) retain the historical height model. The stable root avoids cancellation near low reference heights. The inverse gives a height-crossing age before display rounding. The recovered VasiPred 1.0 code added 0.5 year to its inverse result and then truncated it for a whole-year display. These two steps rounded the age to the nearest whole year. VASIP 2 uses the unrounded inverse in calculations and shows whole years in the readout, as described in Section 2.2. Table 4 summarizes the numerical and file-handling changes.

5.2 Adaptive graph sampling

VasiPred 1.0 joined 16 decadal values with straight lines. VASIP 2 uses a shared adaptive grid for the central curve, bounds, and site-class guides. It includes the reference, prediction, Base, and thinning ages, age 50, the interpolated development limit, and the site-class development ages. Each interval is checked against engine values at its quarter, midpoint, and three-quarter positions. It is bisected if the discrepancy exceeds half the 0.005 m tolerance or its length exceeds five years. The program reports exhaustion of the refinement cap. These checks concern graph interpolation.

The benchmark used version 2.0.20 modules for all sixteen parameter sets over 0–150 years, evaluated on a 0.05-year grid. Reference age was 37.3, with height from the middle SI50 class, $e = 0.1$ m, and a three-SE coefficient grid. Against direct engine evaluation, maximum discrepancies were 0.229857 m for the decadal central-curve polyline and 0.0027943 m across the adaptive plotted quantities, using at most 104 vertices. The tolerance

Table 4: Numerical and file-handling changes relative to VasiPred 1.0.

Finding in the recovered code	Consequence	VASIP 2 calculation or rule
Root subtracts nearly equal terms	Precision loss near low heights	Rationalized positive root and stable log-logistic evaluation
Inverse adds 0.5 year before whole-year truncation	Ages shown only as rounded whole years	Unrounded inverse in all calculations, whole-year readout, hover label to 0.1 year, and no crossing-age column in CSV
81 combinations, including zero-SE repeats	Repeated curve evaluations	Duplicate removal, with 27 unique combined scenarios in one nonzero grid
Bounds clamped to site-class curves	Sensitivity can be hidden	Unclamped bounds and reported outside-range scenario counts
Shared error state and sentinel values	Old or partial results can remain	Pure numerical functions, cleared results, and disabled exports on error
LimitRange replaces values with zero	Extrapolation and zero are confused	Visible predictions with a dashed extrapolation segment
Positional parameter-file matching	Species codes can be mismatched	Checked DEF/LIM import by species code
No horizontal error/bias view	Bound asymmetry is hard to read	Double deviation and bound-sum-bias graphs
Height units only	Small young-age heights obscure relative size	Absolute meter and relative percent deviations

concerns display accuracy; it neither bounds biological model error nor establishes accuracy for every imported parameter set.

5.3 Agreement with the recovered engine

The regression reference was the x64 Release build of `VasiModel.dll`, recovered from the 2020 VasiPred 1.0 project files. It is the native C/C++ computation library of the VasiPred 1.0 solution, which used a C# WinForms interface and the Visual Studio v142 toolset. The recovered files also included `VasiPred.exe` and the DEF/LIM/PAR files. The DLL is a test reference, not an application component. A separate regression compares 4,224 height predictions with its saved direct out-

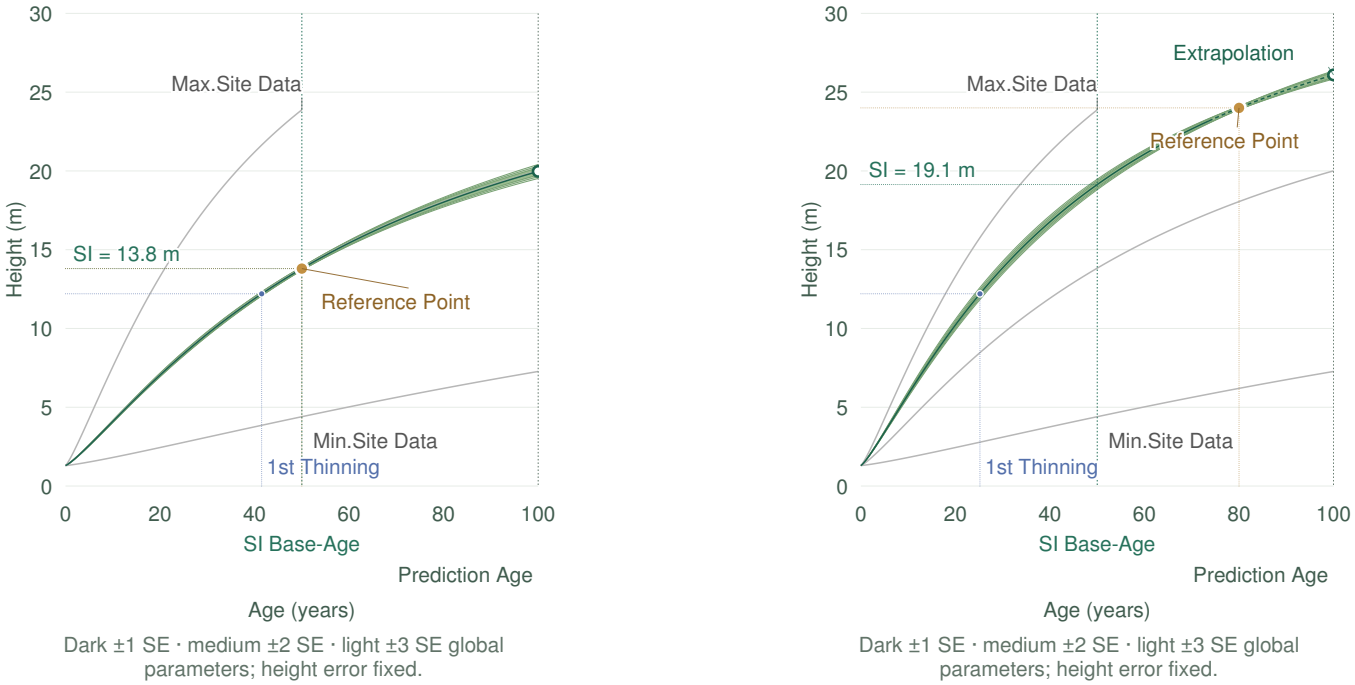

Figure 6: VASIP 2.0.20 graphs in the Light (green) scheme before and after moving the Alberta lodgepole pine reference. Left: 13.8 m at age 50. Right: 24 m at age 80, beyond the 73.6-year interpolated development limit on the dashed Extrapolation segment. Both use $e = 0.1$ m, Base 50, Pred. Age 100, thinning height 12.2 m, and Max.Ht 30 m. Movement changes the local parameter and readouts while the global coefficients and these controls stay fixed. In the right panel, the unlabeled gray curve is the middle SI50 guide; in the left panel it lies under the selected curve.

puts in `tests/legacy-native-output.csv`, read by the engine regression test. It covers sixteen parameter sets, three site classes, eight reference ages, and eleven prediction ages. The maximum absolute difference is 1.4566×10^{-13} m. This supports recovery of the height calculation, including the mathematical extrapolation cases in the grid. It does not imply equal domain rules or envelope displays. The engine test checks reference identity, inverse age, and path invariance (transition invariance) for all sixteen sets at their three SI50 classes and twelve ages from 0 to 500 years. The relative tolerances are 10^{-9} for heights and 10^{-8} for inverse age. Each tolerance is multiplied by the larger of one and the magnitude of the expected value. Invalid inputs, fixed measurement anchors, and nested bounds are also checked.

5.4 Comparison with the 2025 example

Table 5 compares the worked example in Cieszewski and Wawrykowicz (2025) with the VASIP 2.0.20 engine. Both use Alberta lodgepole pine at total height 12 m and breast-height age 50, with a fixed measurement error of 0.10 m. The requested prediction is at age 100, and the requested height is 5 m. The new outer bounds use conditional level 3.

The new central heights are 17.80112243435 m at age 100 and 20.95933156702 m at age 150. Their differences from the printed 17.80 and 20.96 m are +0.00112 and −0.00067 m. These differences are within the rounding of the published numbers. The printed table alone cannot establish equality at greater precision.

At age 100, conditional levels 1, 2, and 3 give combined bounds of 17.58504–18.01730, 17.49029–18.10955, and 17.39428–18.20020 m. At age 150 the corresponding bounds are 20.64972–21.27036, 20.46510–21.45512, and 20.28002–21.63873 m. All three meet 11.90–12.10 m at age 50. Three-SE legacy mode gives the same outer values for this example. The age at 5 m is 15.65898 years. The current readout shows 16 yrs, and the hover label shows 15.7 years (Section 2.2). The printed 16 years in the older paper and the current readout use the same whole-year rounding.

The site index at breast-height age 50 is exactly 12 m here because it is the supplied reference height. This agreement does not remove the Alberta guide-limit difference discussed in Section 6. The current software retains the older bundled guides even when its central height and outer bounds reproduce this published example.

5.5 Lineage and software tests

The build manifest names VasiPred Desktop 3.0.19 as the baseline. It lists SHA-256 hashes of four baseline modules. Three of them, `models.js`, `engine.js`, and `graph-layout.js`, remain byte-identical. The fourth, `app.js`, holds the interface and carries the shell, input precision, display, and reporting changes. These changes leave the three scientific modules unchanged.

Versions 2.0.4–2.0.20 revised startup, graph designs, input precision, pane layout, reports, provenance, citation displays, and the activity log. Version 2.0.17 changed the license. Version 2.0.20 lists the explained graph designs first. These changes left the three scientific modules unchanged; the release notes give the intermediate history.

Table 6 summarizes the build records and the released package. As shipped, 2.0.20 passed 84 of 97 application tests in 11 files, with 13 failures, and all 3 companion-preview tests. Three expectations written for 2.0.19 were updated for 2.0.20: the version label, the inline-script count (14 to 15), and loading the added `graph-presentation.js` before `app.js`. These updates changed the test expectations, not the frozen application. After them, all 100 tests passed with no skips. The 11 application files cover the main suite, parity, startup, appearance, graph designs, deviations, workspace, report panes, activity, data information, and the research report. Engine and interface regressions run within the main suite. The differential record has 78 cases and 138 comparisons, with no failures. It includes all sixteen parameter sets, sources and levels, controls, gestures, imports, projects, and SVG, CSV, and JSON exports. It allows the requested square geometry, changed input precision, and publication metadata. The comparison is

Table 5: The lodgepole pine example from the 2025 paper and VASIP 2.0.20, rounded to the same two-decimal height precision.

	VasiPred 1.0 Printed in 2025			VASIP 2.0.20 Conditional level 3		
Age	Height	Lower	Upper	Height	Lower	Upper
50	12.00	11.90	12.10	12.00	11.90	12.10
100	17.80	17.39	18.20	17.80	17.39	18.20
150	20.96	20.28	21.64	20.96	20.28	21.64

Note: $A_1 = 50$ years, $H_1 = 12$ m, and $e = 0.1$ m. Ages are breast-height years and all heights are total meters. Old values are the published worked example. New bounds use the bundled full-precision coefficients and their SEs.

with Desktop 3.0.19, not a separate test of every browser host. The identical packaged HTML supports using the numerical-kernel results for the shared implementation. It does not establish browser-specific clipboard, download, printing, persistence, or interface behavior. Desktop startup and window tests apply to the WebView2 host.

The startup record restored a Light session. Both plotting axes measured 333.68 px in the documentation window and 462.15 px in the default window. The drag diagnostic passed all checks. Its timings are reported in Section 4. The test suite, the differential records, the saved DLL outputs, and the sampling benchmark are separate from the software supplement. They are in the first author's files and available on request. The released version 2.0.20 package, dated October 17, 2026, has 54 ZIP files and 2,203,281 bytes. All 53 payload checksums listed in `SHA256.json` matched. The remaining ZIP file is `SHA256.json` itself. The archive SHA-256 is `cc9fe77d33f44e04d3e5ce6a039555d26bf2ea2169357475ed666801130a3836`, matching its sidecar.

6 Discussion and Limitations

With the nested bounds, the user can compare growing coefficient perturbations while the measurement-height anchors stay fixed. Retaining inner grids matters because nonlinear extrema need not occur at the outer combinations. The bounds have no verified probability coverage. An exact reference identity does not prove that the measurement is correct. The user chooses its fixed error bound. Marginal SEs do not supply coefficient covariance, residual stand variation, a probability model, or uncertainty in age. A complete prediction interval would need additional statistical information and validation. The deviation and bias displays show bound asymmetry and the growth of the bounds away from their anchors. The displays add no statistical information beyond the input coefficients, their SEs, and the chosen measurement bound.

The bounds here are deterministic sensitivity bounds informed by marginal SEs. In a self-referencing function conditioned on (A_1, H^*), the global coefficients (a, b, c) and the implied q determine the relative height-growth rate and thus propagation away from the reference. The fitted SEs describe coefficient precision under the calibration assumptions. To the extent that lack of fit contributes to the residual variance used to calculate those SEs, its influence is also carried into the coefficient perturbations. The SEs do not identify model-form error separately. Process mode therefore propagates the trajectory-shape uncertainty represented by the fitted coefficients and their SEs without simulating stochastic residual disturbances. The deterministic grid construction does not assign probability coverage to the envelopes.

Statistical assessment of growth-prediction uncertainty uses error budgets, the delta method, Monte Carlo simulation, or empirical error models. Depending on the method, these need coefficient covariance, residual variation, or independent data (Kangas, 1999). Height measurement error can be larger than is often assumed (Jurjević et al., 2020; Wang et al., 2019), and Vertex hypsometer measurements have both systematic and random errors (Bollandsås et al., 2023), so the measurement bound is an input here.

The sixteen bundled parameter sets retain their legacy exploratory status. New Jack Pine is a separate parameterization, with $a = 1.22202$, $b = 24.031$, $c = 1$, and SEs of 0.014599 and 0.637332 for a and b. It is not a twelfth Saskatchewan species. The archive documents it as a $c = 1$ variant of the jack pine fit from March 1995, using the same 116 trees. The white spruce and balsam fir bounds use SEs from runs that did not report convergence, and the aspen bounds use SEs from a run that is not in the archive (Section 2.3). Their bounds should be read with that in mind. Values of ρ above one lie outside the stationary range of a first-order autocorrelation coefficient; the current bounds do not use ρ directly, but they use coefficients and SEs estimated

Table 6: Records for the frozen VASIP 2.0.20 release. Test passes follow the three test-expectation updates described in the text. The package payload checksums all matched.

Check	Recorded result	Scope
Application suite	97 passed	11 files, after the three test-expectation updates
Companion previews	3 passed; 100 in all	No failures or skips in either suite
Differential tests	138 comparisons; no failures	78 cases against Desktop 3.0.19, allowing square geometry, input precision, and publication metadata
Released package payload	53 checksums matched	54 ZIP files, 2,203,281 bytes; archive SHA-256 matches its sidecar
Windows setup file	932,864 bytes	SHA-256 matches its sidecar; 23 files match the released package
Live desktop startup	Ready; square axes	File/View/Export/Import/Help, linked thinning units, SI 13.8 m, and three envelope colors
Fixed default anchors	13.7–13.9 m	All three combined levels at the 13.8 m, age 50 reference with $e = 0.1$ m
Live reference drag	SI changed in 18 frames	All checks true, including the site-line color check and ready flag

in fits that included the lagged-residual term. The maximum ages in LIM are coarse class limits. Interpolating between them is a display rule, rather than a boundary between valid and invalid predictions. The dashed segment and its Extrapolation label mark the interpolated file limit, not the age range of the fitting trees. Appendix B shows that the Alberta development ages are the minimum, mean, and maximum ages of the fitting trees and that several Saskatchewan ages exceed the oldest tree. Neither a dashed curve nor an extrapolation override establishes suitability for management use.

The Alberta guide heights are a known difference from the corrected 2025 paper. This paper identifies Table 2 of Forest Management Note 51 as the source of the uncorrected guide statistics and development ages. The 2025 paper used Forest Management Note 51 heights plus 1.3 m to obtain total heights (Cieszewski and Bella, 1991; Cieszewski and Wawrykowicz, 2025). Its lodgepole pine guides are 25.18, 15.13, and 5.70 m, while the bundle treats 23.88, 13.83, and 4.4 m as total heights. The 1995 file copied the lodgepole pine value, 13.83 m, into the white spruce middle guide. The bundle retains that value. Forest Management Note 51 gives 14.22 m above breast height, or 15.52 m total. These differences affect guides, range warnings, and interpolated development ages. They do not change the coefficients or the central trajectory for a fixed height–age reference. The corrected DEF/LIM import described in Section 2 changes the guides and checks without providing new calibration evidence.

We plan to describe the Desktop 3 screening and fitting workspaces in a later paper, including profiled local parameters and external TSP fits with covariance and serial terms. A further step is to develop and validate covariance-aware or bootstrap bounds. These are plans for later work.

The code hashes identify unchanged modules, the DLL regression checks the recovered height calculation, and the differential and live tests check software behavior. They do not establish biological predictive performance. AI coding assistance does not remove the need for these checks. The records also do not establish installation on a clean computer, every supported Windows configuration, ARM64 operation, screen-reader behavior, or the quality of every printer's PDF output. The differential exports cover SVG, CSV, and JSON, rather than printed PDFs.

7 Availability and Citation

The article page, `https://mcfns.com/index.php/Journal/article/view/342`, lists VASIP 2.0.20 as the setup file `VASIP-2.0.20-Windows-x64-Setup.exe`. Its companion checksum file is named `VASIP-2.0.20-Windows-x64-Setup.exe.sha256`; the checksum is also given below. The setup file has 932,864 bytes. Its SHA-256 is `01c41f695a0b83422169d6fe8e9d6c3c0532d0d6ccb5871a9ccd995799c745a5`, matching the companion file. It installs the 21 application files from the released 2.0.20 archive, `VASIP-2.0.20-Windows-x64.zip` (Table 6), plus its `README.txt` and `RELEASE-NOTES.txt`, byte for byte. These include the Windows application, browser edition, and user guide.

The setup file installs for the current Windows account without administrator rights, by default in `%LOCALAPPDATA%\Programs\VASIP 2`. It checks for 64-bit Windows, .NET Framework 4.8, and the Microsoft Edge WebView2 Runtime, and gives the official download links if a prerequisite is missing. The application's WebView2 DLLs are included; the complete WebView2 Runtime and .NET Framework must be installed separately if needed.

1. Download `VASIP-2.0.20-Windows-x64-Setup.exe` from the article page and its matching `.sha256` file if supplied there.
2. Calculate the setup file's SHA-256 and check that it matches the value above and, when supplied, the companion file.
3. Run the setup file. Windows may warn because it is not code-signed. Use the official download links if a prerequisite is missing.
4. Choose the installation folder and shortcuts. The Start-menu shortcut is on by default; the desktop shortcut is off.
5. Start VASIP 2 from the Start menu.

To remove VASIP 2, use Windows Settings, Apps. Removal keeps the user's sessions in `%LOCALAPPDATA%\VASIP2`. For the standalone browser edition, open `www\app\VASIP-2.html` in the installation folder. The browser edition does not require .NET Framework or the embedded WebView2 host.

The package's `LICENSE.txt` licenses VASIP 2 version 2.0.17 and subsequent author-supplied release material under Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0). This covers the application, documentation, chart and preview material, and parameter-collection presentation. Earlier releases retain their earlier licenses. The Microsoft WebView2 components retain their separate terms and notices in `WebView2-LICENSE.txt` and `WebView2-NOTICE.txt`.

To cite this paper, use Cieszewski and Wawrykowicz (2026), "VASIP 2: Software Update for Visualizing Propagation of Measurement and Process Errors in VASI Based Site Models," MCFNS 18(2). Use the final page range and article link in the journal record. For the software, give VASIP 2, version 2.0.20, publication

package dated October 17, 2026. The frozen package's `CITATION.txt` and citation displays retain article-detail placeholders. Use the authors, title, and final bibliographic details from the journal record when citing the paper.

The 1993 and 1995 Saskatchewan drafts and the TSP fitting archive are in the first author's files and available on request. Code for the model of Cieszewski (2001) used in VASIP 2 and its inverse is given in Appendix A, with expected values for the worked example.

8 Conclusion

VASIP 2.0.20 provides live VASI site curves and nested conditional sensitivity bounds in a separate application for Windows and the browser. Each global-coefficient scenario recomputes the local site parameter, as in VasiPred 1.0. New are the nested levels, the retained inner grids, and the unclamped bounds. All levels keep the same fixed measurement-height anchors. Separate source displays help the user examine measurement and coefficient variation. Absolute and relative double graphs show the signed deviations and bound-sum bias. The program also provides development-limit flags, imports, exports, projects, case reports, and a complete user guide.

Stable evaluation and adaptive sampling improve numerical and graph calculations while retaining the sixteen parameter sets. The DLL regression supports the recovered height equation. The published lodgepole pine heights and outer bounds are reproduced to their printed precision. Interpretation still depends on the legacy parameters, the supplied measurement bound, and the Alberta guide-limit difference.

The verification checks confirm software implementation, live-drag responsiveness, and fidelity of the tested SVG, CSV, and JSON exports; they provide neither new biological validation nor calibrated probability coverage, and VASIP 2 remains a visualization and sensitivity-analysis tool rather than a validated predictive system. The nested envelopes remain deterministic constructions informed by marginal SEs; probabilistic uncertainty assessment requires additional statistical assumptions, coefficient covariance, residual variation, or independent empirical validation appropriate to the method (Kangas, 1999).

Acknowledgments

A.A. Wawrykowicz wrote the original code under the guidance of C.J. Cieszewski: DOS VASIPRED 1.1 in 1993–1995 at the Northern Forestry Centre, Canadian Forest Service, Edmonton, and VasiPred 1.0 in 2019–2020, with a C# interface and a C++ port of the original C computations (Cieszewski and Wawrykowicz, 2025). C.J. Cieszewski adopted the present program from that code and rebuilt VasiPred Desktop 3.x and VASIP 2 using AI coding assistance tools. VASIP 2 re-implements the VasiPred 1.0 height computations in JavaScript (Section 5). The recovered VasiPred 1.0 DLL served as the regression reference in this paper. The Legacy sensitivity mode reproduces the VasiPred 1.0 envelope for the worked example here, except for the clamping described in Section 3.

The Alberta curves (Cieszewski and Bella, 1989, 1991) come from the first author's work with I.E. Bella. The Saskatchewan parameter sets come from the 1995 work under the Canada-Saskatchewan Partnership Agreement in Forestry (PAIF). The bundled files `ALTAVASI` and `VASIP_12` are the historical parameter files of the earlier programs. Their coefficients and SEs are those of the 2025 paper and supplement, except New Jack Pine (Table 1). The supplement also gives the corrected Alberta guides described in Section 2 and does not include New Jack Pine.

The Alberta work was supported in part by the Canada-Alberta Forest Resource Development Agreement. Its data came from the Alberta Forest Service, Weldwood of Canada, and Forestry Canada. The 1993 Saskatchewan work was supported in part by the Canada-Saskatchewan Forest Resource Development Agreement. The 1995 project was supported in part by PAIF. The Saskatchewan data providers were the Saskatchewan Department of Natural Resources (later Environment and Resource Management), Weyerhaeuser Canada, the Canadian Forest Service, and Mistik Management. We thank K. White for customizing SHAZAM and C. Cummins of TSP International for help with the final TSP analysis. I.E. Bella co-authored the 1993 draft and handled administrative aspects of the 1995 project. The second author co-authored the 1993 draft and handled technical aspects of the 1995 project. P. Loseth, P. LeBlanc, and M. Zieba gave feedback.

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References

Bailey, R. L., and J. L. Clutter, 1974. Base-age invariant polymorphic site curves. Forest Science 20(2):155–159. URL `https://doi.org/10.1093/forestscience/20.2.155`.

Bollandsås, O. M., T. H. Eid, and E. A. A. Hansen, 2023. Systematic and random errors of height measurements of individual trees using Vertex hypsometer. MINA fagrapport 87, Norwegian University of Life Sciences (NMBU), Faculty of Environmental Sciences and Natural Resource Management, Ås, Norway. URL https://static02.nmbu.no/mina/publikasjoner/mina_fagrapport/pdf/mif87.pdf.

Cieszewski, C. J., 1988. New Polymorphic Height Growth and Site Index Model for Lodgepole Pine in Alberta. M.F. Major paper. University of British Columbia, Vancouver, B.C.

Cieszewski, C. J., 1995. Base-age invariant polymorphic site index models for eleven timber species in Saskatchewan. Unpublished report manuscript to the Canada–Saskatchewan Partnership Agreement in Forestry. Northern Forestry Centre, Canadian Forest Service, Edmonton, AB.

Cieszewski, C. J., 2001. Three methods of deriving advanced dynamic site equations demonstrated on inland Douglas-fir site curves. Canadian Journal of Forest Research 31(1):165–173. URL https://doi.org/10.1139/x00-132.

Cieszewski, C. J., 2003. Developing a well-behaved dynamic site equation using a modified Hossfeld IV function $Y^3 = (ax^m)/(c+x^{m-1})$, a simplified mixed-model and scant subalpine fir data. Forest Science 49(4):539–554. URL https://doi.org/10.1093/forestscience/49.4.539.

Cieszewski, C. J., and R. L. Bailey, 2000. Generalized algebraic difference approach: Theory based derivation of dynamic site equations with polymorphism and variable asymptotes. Forest Science 46(1):116–126. URL https://doi.org/10.1093/forestscience/46.1.116.

Cieszewski, C. J., and I. E. Bella, 1989. Polymorphic height and site index curves for lodgepole pine in Alberta. Canadian Journal of Forest Research 19(9):1151–1160. URL https://doi.org/10.1139/x89-174.

Cieszewski, C. J., and I. E. Bella, 1991. Polymorphic height and site index curves for the major tree species in Alberta. Forest Management Note 51, Forestry Canada, Northwest Region, Northern Forestry Centre, Edmonton, AB. URL https://publications.gc.ca/site/eng/9.858350/publication.html.

Cieszewski, C. J., I. E. Bella, and A. A. Wawrykowicz, 1993. Site-index height growth curves for the commercial timber species in Saskatchewan. Unpublished draft note, Northern Forestry Centre, Forestry Canada, Edmonton, AB.

Cieszewski, C. J., and A. A. Wawrykowicz, 2025. VASIP: Visualizing measurement and process errors in VASI equations based on Monserud's site index model. Mathematical and Computational Forestry & Natural-Resource Sciences 17(2):41–57. URL https://mcfns.com/index.php/Journal/article/view/17.4.

Cieszewski, C. J., M. Zasada, and M. Strub, 2006. Analysis of different base models and methods of site model derivation for Scots pine. Forest Science 52(2):187–197. URL https://doi.org/10.1093/forestscience/52.2.187.

Hall, B. H., R. Schnake, and C. Cummins, 1992. TSP Version 4.2 Reference Manual. TSP International, Stanford, CA.

Jurjević, L., X. Liang, M. Gašparović, and I. Balenović, 2020. Is field-measured tree height as reliable as believed – Part II, A comparison study of tree height estimates from conventional field measurement and low-cost close-range remote sensing in a deciduous forest. ISPRS Journal of Photogrammetry and Remote Sensing 169:227–241. URL https://doi.org/10.1016/j.isprsjprs.2020.09.014.

Kangas, A. S., 1999. Methods for assessing uncertainty of growth and yield predictions. Canadian Journal of Forest Research 29(9):1357–1364. URL https://doi.org/10.1139/x99-100.

Monserud, R. A., 1984. Height growth and site index curves for inland Douglas-fir based on stem analysis data and forest habitat type. Forest Science 30(4):943–965. URL https://doi.org/10.1093/forestscience/30.4.943.

Wang, Y., M. Lehtomäki, X. Liang, J. Pyörälä, A. Kukko, A. Jaakkola, J. Liu, Z. Feng, R. Chen, and J. Hyyppä, 2019. Is field-measured tree height as reliable as believed – A comparison study of tree height estimates from field measurement, airborne laser scanning and terrestrial laser scanning in a boreal forest. ISPRS Journal of Photogrammetry and Remote Sensing 147:132–145. URL https://doi.org/10.1016/j.isprsjprs.2018.11.008.

White, K. J., 1978. A general computer program for econometric methods—SHAZAM. Econometrica 46(1):239–240.

Appendix A: Code for the Cieszewski (2001) model used in VASIP 2 as the site dependent height model

The listings evaluate the model of Cieszewski (2001) (Eq. 21 applied to heights above the base height) and its inverse, Eqs. (1)–(3), using the file parameters a, b, c, and h_b and the rationalized root. They are compact examples; the application additionally uses log-domain height evaluation and input checks. The conversion is $\delta = a$, $\kappa = 2b$, and $\eta = bc^{-a}$; for readers with Eq. 21 parameters, $a = \delta$, $b = \kappa/2$, and $c = (b/\eta)^{1/a}$. For the Alberta lodgepole pine example in Section 5.4, the bundled file values are $a = 1.203724$, $b = 1947.494784$, $c = 50$, and $h_b = 1.3$ m, with $H_1 = 12$ m and $A_1 = 50$ years. The expected values are $H(100) = 17.80112243435$ m, $H(150) = 20.95933156702$ m, and $A_T = 15.65898$ years at $H_T = 5$ m.

These short examples require positive a, b, c, A_1, and A, with $H_1 > h_b$ and $h_b < H_T < h_b + q + d$. At age zero the model height is h_b, and at target height h_b the inverse age is zero. In the worked example $h_1 - d < 0$, so the cancellation-free branch is used. In SAS and TSP, which ignore letter case, `h1x` denotes h_1 and `Apred` denotes A, and `hTx` denotes h_T to keep them distinct from `H1`, `a`, and `HT`. TSP uses `cref` for c because `C` is reserved and cannot be declared as a parameter. In Excel, enter each value or formula in the cell shown (a formula continued on the next line belongs to the same cell; B12 and C12 hold the two prediction ages).

Excel

```
Cell (name): contents
B1 (a): 1.203724
B2 (b): 1947.494784
B3 (c): 50
B4 (hb): 1.3
B5 (H1): 12
B6 (A1): 50
B7 (HT): 5
B8 (d): =B2*B3^(-B1)
B9 (h1): =B5-B4
B10 (R): =SQRT((B9-B8)^2+
  4*B2*B9*B6^(-B1))
B11 (q): =IF(B9-B8<0,
  2*B2*B9*B6^(-B1)/(B10-(B9-B8)),
  (B9-B8+B10)/2)
B12 (A): 100
C12 (A): 150
B13 (H): =$B$4+$B$11*($B$11+$B$8)/
  ($B$11+$B$2*B12^(-$B$1))
C13 (H): =$B$4+$B$11*($B$11+$B$8)/
  ($B$11+$B$2*C12^(-$B$1))
B14 (hT): =B7-B4
B15 (AT): =(B2*B14/
  (B11*(B11+B8-B14)))^(1/B1)
```

TSP

```
PARAM a 1.203724, b 1947.494784, cref 50;
PARAM hb 1.3, H1 12, A1 50, HT 5;
SMPL 1 2;
GENR Apred=100;
SMPL 2 2;
GENR Apred=150;
SMPL 1 2;
SET d=b*cref**(-a);
SET h1x=H1-hb;
SET R=SQRT((h1x-d)**2+4*b*h1x*A1**(-a));
IF h1x-d < 0; THEN;
SET q=2*b*h1x*A1**(-a)/(R-(h1x-d));
ELSE;
SET q=(h1x-d+R)/2;
GENR H=hb+q*(q+d)/(q+b*Apred**(-a));
SET hTx=HT-hb;
GENR AT=(b*hTx/(q*(q+d-hTx)))**(1/a);
PRINT Apred H AT;
END;
```

SAS

```
data _null_;
  a=1.203724; b=1947.494784; c=50;
  hb=1.3; H1=12; A1=50; HT=5;
  d=b*c**(-a); h1x=H1-hb;
  R=sqrt((h1x-d)**2+4*b*h1x*A1**(-a));
  if h1x-d < 0 then
    q=2*b*h1x*A1**(-a)/(R-(h1x-d));
  else q=(h1x-d+R)/2;
  hTx=HT-hb;
  AT=(b*hTx/(q*(q+d-hTx)))**(1/a);
  do Apred=100,150;
    H=hb+q*(q+d)/(q+b*Apred**(-a));
    put Apred= H=20.11 AT=20.11;
  end;
run;
```

R

```
height_equation <- function(a,b,c,hb,H1,A1,A,HT) {
  d <- b*c^(-a); h1 <- H1-hb
  R <- sqrt((h1-d)^2+4*b*h1*A1^(-a))
  if (h1-d < 0) {
    q <- 2*b*h1*A1^(-a)/(R-(h1-d))
  } else {
    q <- (h1-d+R)/2
  }
  H <- hb+q*(q+d)/(q+b*A^(-a))
  hT <- HT-hb
  AT <- (b*hT/(q*(q+d-hT)))^(1/a)
  list(H=H, AT=AT)
}
height_equation(1.203724,1947.494784,50,1.3,
                12,50,c(100,150),5)
```

C++

```
#include <cmath>
#include <iomanip>
#include <iostream>
struct Result { double H, AT; };
Result height_equation(double a, double b,
    double c, double hb, double H1, double A1,
    double A, double HT) {
  using std::pow;
  double d=b*pow(c,-a), h1=H1-hb;
  double R=std::sqrt(pow(h1-d,2)
                   +4*b*h1*pow(A1,-a));
  double q;
  if (h1-d < 0)
    q=2*b*h1*pow(A1,-a)/(R-(h1-d));
  else q=(h1-d+R)/2;
  double H=hb+q*(q+d)/(q+b*pow(A,-a));
  double hT=HT-hb;
  double AT=pow(b*hT/(q*(q+d-hT)),1/a);
  return {H,AT};
}
int main() {
  double ages[]={100,150};
  for (double A : ages) {
    auto v=height_equation(1.203724,1947.494784,
                           50,1.3,12,50,A,5);
    std::cout << std::fixed << std::setprecision(11)
              << A << " " << v.H << " " << v.AT
              << '\n';
  }
}
```

Appendix B: Historical calibration records

Alberta

Forest Management Note 51 (Cieszewski and Bella, 1991) used stem analyses of dominant and codominant trees from the Alberta Forest Service, Weldwood of Canada (Hinton Division), and Forestry Canada. Trees needed non-decreasing heights and ages, at least three valid measurements above breast height, and a breast-height age of at least 50 years. Aspen also had to reach breast height by age five. Further screening removed low site indices, high ages at the first section, and early suppression. The retained counts were 1,163 lodgepole pine, 698 white spruce, 282 black spruce, and 276 trembling aspen. Heights were interpolated at decadal ages. Nonlinear least squares used the preceding height and age in non-overlapping decadal periods. The fits used customized SIMPLEX software and were rerun in SAS for further statistics.

In the note's coefficient notation, the file parameterization writes a as one plus α and b as twenty times β. The SE of a is the SE of α, and the SE of b is twenty times the SE of β. This converts the height form of the note's FORTRAN listing to Eq. (2) with $c = 50$. The printed note and file values differ at some digits. We retain the file values. The note's summary of the fitting data gives the source statistics for the SI50 guides: maximum, mean, and minimum SI above breast height. Some stored guides differ from these statistics. The development ages are that summary's minimum, mean rounded to five years, and maximum breast-height ages. The note also prints the interactive FORTRAN program that preceded the later programs.

Saskatchewan

The draft note by Cieszewski et al. (1993) used SHAZAM (White, 1978). Its stem analyses came from the Saskatchewan Department of Natural Resources and Weyerhaeuser Canada, Prince Albert. The first author's later report draft (Cieszewski, 1995) is the internal 1995 report cited by Cieszewski and Wawrykowicz (2025). Its data came from the Saskatchewan Department of Environment and Resource Management, Weyerhaeuser Canada (white spruce), and Mistik Management (trembling aspen). Both drafts used the three initial screening rules above, then visual screening for early suppression and top damage, and interpolation of decadal heights. The report's height part equals Eq. (2) with $c = 50$ and $h_b = 1.3$ m. Its C is d of Eq. (1), and its *HRoot* is twice q plus d.

The report describes refitting the SHAZAM models in TSP (Hall et al., 1992). The final runs in `SASK_TSP.ZIP` are dated April 1995 and used TSP 4.2B. Each tree had a local parameter. The fitted lagged-residual term used ρ times the preceding decadal residual of the same tree. The runs fitted common parameters, then tree parameters, then common parameters again, and finally all parameters simultaneously by least squares. They used analytical derivatives. Ten of the eleven coefficient pairs and all their SEs are identical to these final estimates. Trembling aspen is the exception. Its file values come from a run that is not in the archive. TSP did not report convergence within the 40-iteration limit for the final white spruce and balsam fir runs or for the archived aspen run. The other final runs converged. The eleven archived fits used 547 trees and 4,725 observations. The report lists 64 screened aspen trees, while the archive run has 31. It lists 38 balsam poplar trees, while the final run has 37.

The High and Low SI50 guides equal the maximum and minimum tree SI in the final data for nine species. Balsam poplar's 23.28/16.05/7.01 m guides are the maximum, mean, and minimum of an earlier 38-tree data file in the archive (March 1995). Its final data for 37 trees have a mean of 16.29 m and minimum of 11.03 m. Middle guides are within 0.01 m of the mean tree SI for lodgepole pine, white elm, black spruce, white birch, balsam fir, and tamarack larch. The differences for jack pine, white spruce, and trembling aspen are 0.02–0.06 m. Manitoba maple's middle and low guides, 14.02 and 10.51 m, differ from the data mean and minimum, 13.59 and 10.21 m. The archive does not explain them. The High development age equals the last measured age of the highest-SI tree for eight species. Several Middle and Low values, 275, 268, 245, 200, and 194 years, exceed the oldest tree in their data.

New Jack Pine matches the archive run `PJP22` of March 5, 1995. It used the same 116 jack pine trees and 904 observations, with $c = 1$, so $d = b$. Each tree's local fitting parameter was its height above breast height at age 50, without the measured-SI offset used in the final runs. The sum of squared residuals was 105.743, against 119.903 for the final $c = 50$ jack pine run. Table 2 summarizes these source fits.