ISSN 1946-7664. MCFNS 2023
Vol. 15, Issue 2, pp. 33–41
Mathematical and Computational Forestry & Natural-Resource Sciences

AVAILABLE ONLINE AT HTTP://MCFNS.COM

Submitted: Jun. 7, 2022
Accepted: Oct. 30, 2023
Published: Oct. 30, 2023
Last Correction: Nov. 14, 2023

ALTERNATIVE METHOD FOR DETERMINING TRUNK DIAMETER AT DIFFERENT HEIGHTS ON A STANDING TREE

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Abstract. Reliably determining trunk volume of a growing tree and accurately measuring changing diameters along the trunk at different heights are important data to foresters world-wide. Existing methods for determining trunk diameter of a growing tree assume the horizontal cross-section is a circle. However, to an observer standing beside a tree and looking upwards along the trunk, the imaginary cross-section of the trunk does not look like a circle but appears elliptical. As the observer stands closer to the tree and the higher the established point where the diameter of the trunk is measured, the elliptical shape of the cross-section becomes more pronounced. Conversely, the smaller the tree and the farther the observer stands from the tree, the imaginary cross-section of the trunk becomes more circular. In this paper we describe a method that makes it possible to accurately determine the diameter of a trunk regardless of the above two distance factors. Using the mathematical parameters of an ellipse, the objective of this study was to provide an accurate method to calculate the diameter at any height on the trunk.

Keywords: Calipers; DBH; diameter-at-breast-height; forest measurement and inventory; laser mensuration; error estimation.

1 Introduction

Accuracy of trunk volume determination is linked to the accuracy of trunk diameter and tree height measurements (Elzinga et al. 2005). There are several methods for determining the volume of a trunk. One method is the complex Smalian formula that uses diameters at different heights along the trunk (Kershaw et al. 2017). Using tools such as clinometers, calipers and hypsometers, direct measurements are taken to determine diameters at different trunk heights (van Laar and Akça 1997; Husch et al. 2003; Luoma et al. 2017). Remote-sensing girth measurement methods are also used (Bauwens et al. 2016; Ahola et al. 2021). In recent years, measurements using laser devices and 2D scanning have been used to measure trunk diameters of growing trees (Liang et al. 2014a, c; Mokroš et al. 2018; Čerňava et al. 2019; Fan et al. 2019). Photogrammetry (Liang et al. 2014b; Eliopoulos et al. 2020) and mobile phone applications are also used to measure trunk diameters (Pratihast et al. 2014; Vastaranta et al. 2015; Molinier et al. 2016).

In the Evo region of southern Finland, Luoma et al. (2017) used clinometers in conjunction with calipers to measure trunk diameters to investigate accuracy of direct measurements of 319 trees at diameter-at-breast-height (dbh). They found no significant differences in accuracy of independent trunk dbh measurements. The standard deviation in dbh measurements was only 1.5%. As a result, they support the idea of using traditional measuring tools as a substitute benchmark for modern trunk diameter electronic measurement methods.

Guillemette and Lambert (2009) investigated accuracy of dbh trunk measurements using tape measures and calipers. They noted that differences in these measurements varied from 0 to 11 mm. However, the difference in calculations of accumulated trunk volume per a specific forest area was significant, amounting to $10.5 m^3 \cdot ha^{-1}$ within an average stand volume of

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$169m^3 \cdot ha^{-1}$. Therefore, it was recommended that data obtained with dendrometers not be mixed.

Currently, the use of laser scanning is increasing when conducting forest inventories. In one study, Bauwens et al. (2016) compared a 'handheld mobile laser scanner' (HMLS) with two 'terrestrial laser scanning' (TLS) methods (single scan: SS; multiple scan: MS) to evaluate forest inventory. They found that SS was better suited for land delineation of forest patches, whereas MS provided better results for describing the upper part of the canopy. For 91% of trees with dbh $>$10 cm, a vertical transect of 1.3 m per trunk was scanned with a HMLS, which gave the best results for dbh estimates (error 0.08 cm; standard error 1.11 cm) compared with partially scanned trees for SS and 42% of fully scanned trees for MS.

Mokroš et al. (2018) effectively used a circle-fitting algorithm to estimate dbh. Bienert et al. (2007) analyzed forest terrestrial laser scanner 'point clouds' for tree detection and estimating trunk diameters. However, in some studies 'photogrammetric point cloud' (PPC) mean trunk diameter measurements were consistently greater compared with actual mean ground diameters, indicating existence of bias (Molinier et al. 2016; Fang and Strimbu 2017).

As previously noted, several methods are currently available that explain procedures to measure trunk diameters from specific distances (Fang and Strimbu 2017; Eliopoulos et al. 2020; Ahola et al. 2021). However, when using these methods, it is assumed that the cross-section of a trunk is always circular, which is only possible in two cases, either by measuring dbh or by directly measuring the trunk diameter at the height of a measuring device. If the cross-section is perpendicular to the longitudinal axis, it is a circle. In all other cases, the cross-section of the trunk is an ellipse. Therefore, theoretical assumptions suitable for measuring the trunk diameter only at dbh, or at the height of measuring devices for other diameters will give distorted results.

In this study we describe an alternative method for accurately calculating trunk diameters at different heights above the ground surface. The idea of this method is that when scanned at different trunk heights from the ground, the cross-section of the trunk does not look like a circle but takes the shape of an ellipse, while the horizontal cross-section diameter appears as a circle (Ringdahl et al. 2013; Pratihast et al. 2014). Therefore, determination of trunk diameters at different heights by traditional methods leads to displacement of obtained results from accurate dimensions of the diameter at any specified height on the trunk (Molinier et al. 2016). By using ellipse parameters, our proposed method makes it possible to determine the diameter of the circular cross-section at different trunk heights with a high degree of accuracy. Measurement errors or differences in direct measurements compared with calculated diameters are mostly the result of small divergences in measuring tape placement from the plane perpendicular to the trunk axis (van Laar and Akça 1997).

2 Materials and Methods

2.1 Study Area

Studies were conducted during July through August 2022 near the city of Samukh, Republic of Azerbaijan (40°45' 21" N – 46°25' 16" E) on a forest site located within a 3.6 ha research area (Figures 1 and 2).

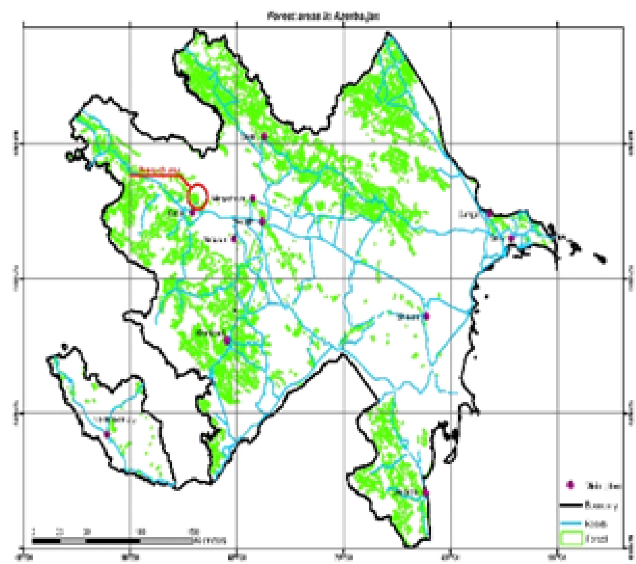


Figure 1: The study location near the city of Samukh is circled in red.

The two main forest tree species measured were even-aged, single canopy 'long-stemmed' oak (*Quercus longipes* Hu 1951 [synonym for 'ring-cupped' oak, *Q. glauca* Thunberg 1784]) and black locust (*Robinia pseudoacacia* L [local common name: white acacia]). Average taxonomic parameters: oak, age 40 years, height 14 m, dbh 28 cm; black locust, height 12 m, dbh 20 cm.

2.2 Terrestrial Laser Mensuration

A $50m \times 200m$ rectangular tree measurement site was established within the research area (Figure 2). A Leica TC407 laser transit 'total station tachometer' (TC407), Masser BT Caliper and measuring tape were used for taking measurements. The TC407 was set-up at 1.58-m high at the laser beam emitter level with the tripod firmly on the ground surface at point B. The TC407 was placed at pre-set distances of 4, 6, 10, and 15 m from the trunk (distance BK; Figure 3).

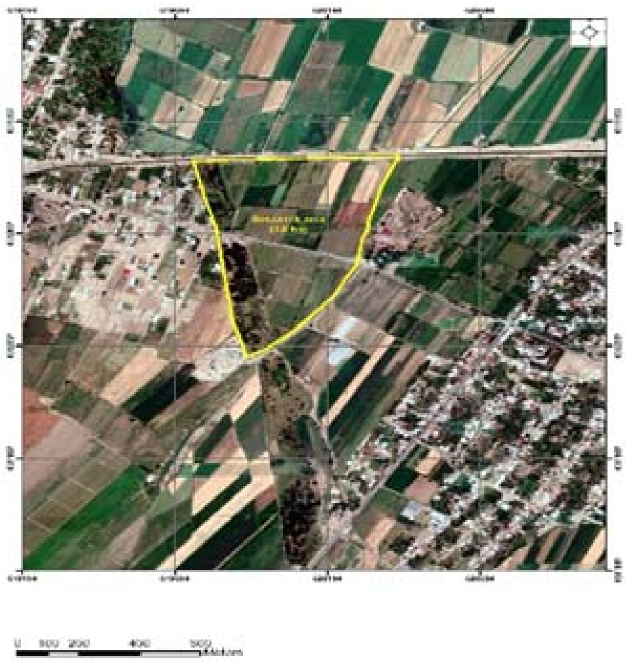
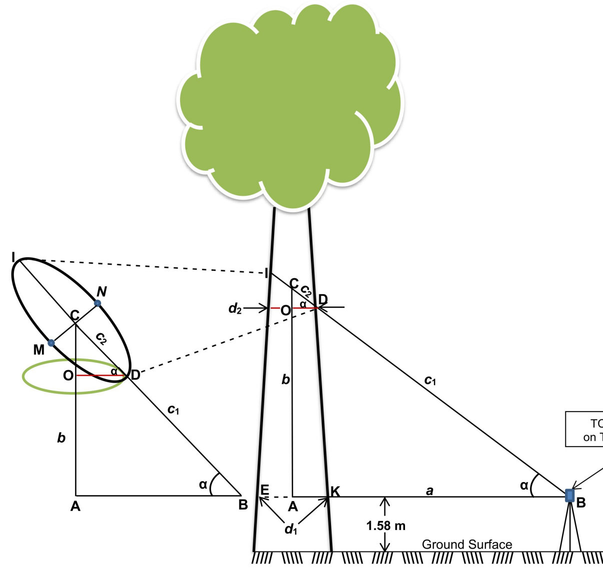


Figure 2: Research area (3.6 ha) within yellow lines. Tree measurement forest site: 50- x 200-m green rectangle.



$AB \quad = a + d_1/2$
$AC \quad = b$
AK (radius) $= d_1/2$
$BD \quad = c_1$
$BK \quad = a$
$CD \quad = c_2$
$EK \quad = d_1$
$OD \cdot 2^* \quad = d_2$

Note: D—Direct measurement point of tree diameter compared with calculated diameter values at point D.

*There are two OD: direct measurement and calculated.

Figure 3: Trunk measurement parameters.

For each laser measurement the TC407 set-up location was chosen with a clear view of each trunk scanned. On both sides of the trunk where the laser beam impacted the centerline of the trunk surface, the diameter of the trunk was directly measured twice at a 45°angle from each side of the laser beam, ensuring a 90°angle between the two full diameter measurements. The average diameter of each trunk was then calculated.

Using a ladder, the Masser BT Caliper was used to directly measure the diameter (d_2) of the trunk at point D above the TC407 horizontal laser line height so that the diameter location exactly matched the laser beam impact point on the trunk (Figure 3). It is assumed that if directly measured points along the trunk obtained accurate diameter results, then measurements at other points along the trunk would also be accurate.

By adding the radius ($d_1 \cdot 2^{-1}$; AK) of the trunk measured at the height of laser line BK we find the total length of line BA ($BA = a + d_1 \cdot 2^{-1}$). The TC407 was used to measure the distance from B to D (BD) and to determine the vertical angle α to different measurement points (D) along the trunk. Because triangles BAC and DOC are similar 'right' triangles, and that side DC is a continuation of side BD and sides AB and OD are parallel, then angles at D and B are equal ($\angle ABD = \angle ODC = \angle\alpha$).

By measuring the angle at point B, we know the angle at point D. Since BC is the hypotenuse of a right triangle and BA is the adjacent side of BAC, then BC >BA. Therefore, because triangles BAC and DOC are similar and OD is parallel to BA, then DC >DO and OD is a radius of a circle at point D. As angle α at point D increases, the slanted cross-section DI becomes increasingly elliptical, with $DI \cdot 2^{-1}$ equal to one-half the major axis of the ellipse (CD; c_2). For triangle BAC, length $BC = AB \cdot cos(\alpha)^{-1}$. Therefore, BC–BD = CD. We can calculate the trunk radius at point D: $OD = CD \cdot \cos(\alpha)$, hence the true diameter of the trunk at height $D = OD \cdot 2$, and at any other point on the trunk by the same method.

3 Results

3.1 Analysis of Diameter Measurements

The study was conducted using the following parameters: trunk diameter (d_1) measured at TC407 horizontal laser beam height (1.58 m); distance BK ($= a$) from the TC407 (B) to the trunk surface (K); measure trunk diameters at different trunk heights (d_2; Point D); distance c_1 from the TC407 to the point of measurement of trunk diameters at different trunk heights; and angle α to the point of measurement of trunk diameters (d_2) at different trunk heights (AO + 1.58 m); and angle α to the trunk diameter point of measurement (D).

Based on these measurements, lengths such as distance AC (= b) from the center of the circle-shaped diameter (A) of the trunk at TC407 level to the center of the slanted ellipse-shaped section (C), length of one-half of the ellipse major axis CD (c_2), and true trunk diameter (d_2) at point D were calculated (Figure 3; Table 1 and 2).

We conducted statistical analyses of the results of calculated and directly measured trunk diameter values (Table 1 and 2). Trunk diameters of 32 oak and 18 black locust trees were measured from distances of 4, 6, 10 and 15 m from the TC407 at the laser beam height of 1.58 m above the ground surface (Figure 3). Trunk diameters ranged from 0.13 to 0.56 m for oak, and 0.142 to 0.493 m for black locust. Variation in the heights on trunks above TC407 laser beam height (1.58 m) where diameters were measured ranged from 0.0165 to 2.9055 m for oak, and 0.2043 to 2.0340 m for black locust. The smallest and largest differences between calculated and directly measured diameters were –0.0003 m and +0.0458 m, and +0.0002 m and –0.0338 m, for oak and black locust, respectively (Table 1 and 2).

Accuracy of the proposed method for determining the diameter at any trunk height was assessed by comparing directly measured with calculated diameters, then calculating the error bias, relative bias, root mean square error (RMSE) and relative RMSE, as defined by the following equations (Sun et al. 2020; Table 3):

1. Bias:

$$\text{Bias} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y}_r)$$

2. Relative Bias (rBias):

$$\text{rBias} = \frac{\text{Bias}}{\bar{y}_r} \cdot 100\%$$

3. Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\frac{\sum(y_i - \bar{y}_r)^2}{n}}$$

4. Relative RMSE (rRMSE):

$$\text{rRMSE} = \frac{\text{RMSE}}{\bar{y}_r} \cdot 100\%$$

Directly measured diameters compared with calculated diameters for 50 trees were nearly equal. There were no significant differences between calculated and directly measured trunk diameters, validating that our methods are accurate ($\alpha \leq 0.05$; Table 1 and 2). Bias and rBias between the calculated and directly measured data are not significant. Bias varied from -0.01249 to 0.01246, and rBias from -4.483 to 4.840%. A small exception is noted in the fourth row (15 m), where the discrepancy between directly measured and calculated data was Bias (0.01246 m), rBias (4.840%), RMSE (0.03511), and rRMSE (13.64%), which in some cases can be explained by comparisons with values in rows 4, 6, and 10 m that represent relatively large inaccuracies of $\approx 1cm$ for some measurements.

Statistical analysis showed a high correlation coefficient ($r = 0.9863$; $r^2 = 0.9728$) between calculated and directly measured trunk diameters, which is reflected as a linear regression equation: $y = 0.9584x+0.0128$, where x axis = calculated trunk diameter, and y axis = directly measured trunk diameter (Bewick et al. 2003; Abdi 2007; Figure 4).

Random errors, **m**, of parameters $a = 0.9584$, $b = 0.0128$, and correlation coefficient $r_{xy} = 0.9863$ are calculated as follows:

$$\mathbf{m}_a = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n-2} \cdot \frac{\sum x_i^2}{n(\sum(x_i - \hat{x}_i)^2}}$$
$$= \sqrt{\frac{0.0136}{50-2} \cdot \frac{0.7339}{50 \cdot 0.5268}} \approx 0.0063 \qquad \text{[free term]}$$

$$\mathbf{m}_b = \sqrt{\frac{1}{\sum(x_i - \hat{x}_i)^2} \cdot \frac{\sum(y_i - \hat{y}_i)^2}{n-2}}$$
$$= \sqrt{\frac{1}{0.5268} \cdot \frac{0.0136}{50-2}} \approx 0.0232 \quad \text{[regression coefficient]}$$

$$\mathbf{m}_{r_{xy}} = \sqrt{\frac{1 - r_{xy}^2}{n-2}}$$
$$= \sqrt{\frac{1 - 0.9863^2}{50-2}} \approx 0.0238 \quad \text{[correlation coefficient]}$$

F [Fisher's test]

$$F_{\text{calculated}} = \frac{r_{xy}^2}{1 - r_{xy}^2} \cdot (n-2)$$
$$= \frac{0.9728}{1 - 0.9728} \cdot (50-2) \approx 1716.71 > F_{\text{tab}} \approx 4.03,$$

where: $k_1 = 1;\ k_2 = 50 - 2 = 48;\ \alpha = 0.05$.

Student's t-test

$$t_b = \frac{a}{m_b} = \frac{0.9584}{0.0232} \approx 41.361 > t_{\text{tab}} \approx 2.02;$$
$$t_a = \frac{b}{m_a} = \frac{0.0128}{0.0063} \approx 2.032 > t_{\text{tab}} \approx 2.02.$$
$$t_{r_{xy}} = \frac{r_{xy}}{m_{r_{xy}}} = \frac{0.9863}{0.0238} \approx 41.361 > t_{\text{tab}} \approx 2.02;$$
$$df = n - 2 = 48; \quad \alpha = 0.05.$$

Table 1: Comparison of calculated and direct measurement trunk diameters (d_2) of Oaks at different heights above TC407 height (1.58 m).

Mean diameter from two direct perpendicular measurements at 1.58-m height, m	Height of direct measurement point(s) D above TC407 height (1.58 m), m	Calculated diameters (d2) at point(s) D, m (x)	Direct measurement diameters (d2) at point(s) D, m (y)	Difference between x and y diameters, m $(x-y)^a$	$(x-y)^2$
Distance to trunk from the TC407 = 4 m, 13 measured trees					
0.215	0.3321	0.1834	0.1819	0.0015	0.00000
0.255	0.1436	0.2219	0.2022	0.0203	0.00041
0.250	0.4368	0.1989	0.2219	-0.0230	0.00053
0.145	0.4119	0.1244	0.1029	0.0215	0.00046
0.142	0.6889	0.1232	0.1370	-0.0138	0.00019
0.137	1.2645	0.0764	0.0920	-0.0157	0.00024
0.130	1.5706	0.0746	0.0850	-0.0104	0.00010
0.251	0.3487	0.1993	0.1821	0.0172	0.00030
0.275	0.5726	0.2437	0.2590	-0.0153	0.00023
0.275	1.1199	0.1989	0.2250	-0.0261	0.00068
0.340	0.5705	0.2180	0.2390	-0.0210	0.00044
0.350	0.3439	0.2366	0.2487	-0.0121	0.00015
0.323	0.8986	0.2151	0.1918	0.0233	0.00054
		Sum:	**2.3685**	**-0.0938**	**0.00428**
Distance to trunk from the TC407 = 6 m, 5 measured trees					
0.525	0.5386	0.5191	0.5198	-0.0007	0.00000
0.240	1.0637	0.1790	0.1989	-0.0199	0.00040
0.220	0.0165	0.1738	0.1950	-0.0212	0.00045
0.510	1.4756	0.4818	0.4731	0.0077	0.00006
0.243	0.2412	0.2358	0.2212	0.0146	0.00021
		Sum:	**1.6080**	**-0.0195**	**0.00112**
Distance to trunk from the TC407 = 10 m, 7 measured trees					
0.240	2.9055	0.2019	0.2240	-0.0221	0.00049
0.300	0.3433	0.2798	0.2340	0.0458	0.00210
0.560	0.3869	0.5296	0.5299	-0.0003	0.00000
0.260	0.4221	0.2538	0.2563	-0.0025	0.00001
0.270	0.7816	0.2552	0.2681	-0.0129	0.00017
0.272	1.7444	0.1991	0.2323	-0.0332	0.00110
0.418	0.6655	0.3972	0.4056	-0.0084	0.00007
		Sum:	**2.1502**	**-0.0794**	**0.00392**
Distance to trunk from the TC407 = 15 m, 7 measured trees					
0.374	0.2810	0.3600	0.3520	0.0080	0.00006
0.258	0.4465	0.2538	0.2519	0.0019	0.00000
0.245	0.9737	0.2336	0.2115	0.1221	0.01491
0.390	1.1562	0.3398	0.3175	0.0223	0.00050
0.380	0.5092	0.3798	0.3678	0.0120	0.00014
0.322	0.8886	0.3090	0.2810	0.0280	0.00078
0.352	1.4542	0.3205	0.3367	-0.0162	0.00026
		Sum:	**2.1184**	**0.1943**	**0.01667**

[a]Across rows, there are no significant differences between calculated and directly measured trunk diameters. $\alpha \leq 0.05$.

Random errors parameters of the regression equation ($y = 0.9584x + 0.0128$), and correlation coefficient ($r = 0.9863$; $r^2 = 0.9728$) were used for F and t-tests. Results provide a high level of correlation between calculated and directly measured trunk diameters. Therefore, calculated diameters determined using a TC407 or similar measurement devices are accurate and can be used

Table 2: Comparison of calculated and direct measurement trunk diameters (d_2) of Black Locust (White Acacia) at different heights above TC407 height (1.58 m).

Mean diameter from two direct perpendicular measurements at 1.58-m height, m	Height of direct measurement point(s) D above TC407 height (1.58 m), m	Calculated diameters (d2) at point(s) D, m (x)	Direct measurement diameters (d2) at point(s) D, m (y)	Difference between x and y diameters, m $(x-y)^a$	$(x-y)^2$
Distance to trunk from the TC407 = 4 m, 1 measured tree					
0.223	0.2344	0.2157	0.2111	**0.0046**	**0.00002**
Distance to trunk from the TC407 = 6 m, 4 measured trees					
0.273	0.3013	0.2397	0.2422	-0.0025	0.00001
0.493	0.3467	0.4792	0.4851	-0.0059	0.00003
0.324	0.2072	0.3198	0.3317	-0.0119	0.00014
0.355	0.2077	0.3061	0.3012	0.0049	0.00002
		Sum:	**1.3602**	**-0.0154**	**0.00021**
Distance to trunk from the TC407 = 10 m, 6 measured trees					
0.255	0.5212	0.2337	0.2449	-0.0112	0.00013
0.240	1.4613	0.1880	0.2226	-0.0338	0.00114
0.365	0.5291	0.3296	0.3317	-0.0021	0.00000
0.270	0.4318	0.2598	0.2699	-0.0101	0.00010
0.235	0.8895	0.2270	0.2115	0.0156	0.00024
0.228	1.7265	0.1636	0.1893	-0.0257	0.00066
		Sum:	**1.4699**	**-0.0829**	**0.00228**
Distance to trunk from the TC407 = 15 m, 7 measured trees					
0.410	0.2043	0.3528	0.3414	0.0114	0.00013
0.241	2.0430	0.2319	0.2317	0.0002	0.00000
0.268	0.7914	0.2097	0.2091	0.0006	0.00000
0.272	0.2367	0.2699	0.2712	-0.0013	0.00000
0.164	0.2748	0.1600	0.1615	-0.0015	0.00000
0.160	0.7531	0.1398	0.1322	0.0076	0.00006
0.142	1.1228	0.1197	0.1399	-0.0202	0.00041
		Sum:	**1.4870**	**-0.0198**	**0.00060**

[a]Across rows, there are no significant differences between calculated and directly measured trunk diameters. $\alpha \leq 0.05$.

Table 3: Comparisons of the calculated and directly measured diameters at any height on a trunk were obtained using a standard measuring tape, Masser BT caliper and TC407.

Distance from the TC407 to the trunk, m	Bias, m	Relative bias, % rBias	Root mean squared error, RMSE	Relative root mean squared error, % rRMSE
4	-0.007029	-3.814	0.0175	9.510
6	-0.003878	-1.176	0.0121	3.679
10	-0.012490	-4.483	0.0218	7.840
15	0.012460	4.840	0.0351	13.640

for diameter measurements instead of direct measurement methods.

4 Discussion

Accurate measurement of diameters at any trunk height is important not only for measuring trunk volume, but also in terms of designing and testing new, more modern measuring equipment. The closer the measuring equipment is placed to the trunk and the higher on the trunk you need to measure the diameter, the greater the measurement error. The elliptical shape of the cross-section at the height of the measuring equipment coincides with a circle. However, the major axis of the ellipse is not the actual diameter of the trunk at a point above dbh or at the height of the measuring

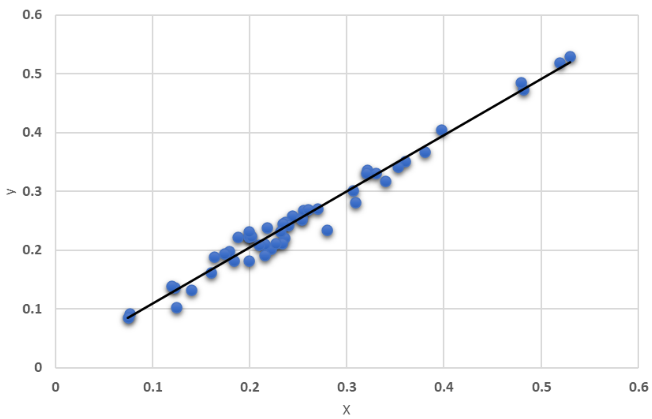


Figure 4: Correlation between calculated and directly measured trunk diameters.

equipment. This study provides a method by which ellipse data can be used to calculate the accurate diameter of a trunk at any height.

Therefore, using the distance to the trunk, measuring the vertical angle to the measurement point of the trunk diameter, and using known trigonometric formulas, the mathematical problem of finding the accurate diameter of the trunk from the measurement data of the ellipse parameters is solved. To study the validity of the results, trunk diameters were directly measured at different heights above the TC407 height and at four different distances from the trunks, providing multiple data points and comparability of results.

Results of statistical analysis showed a high degree of agreement between calculated and directly measured diameters. Bias and rBias were not significant, and any discrepancies were mostly caused by minor variable direct measurements with calipers. RMSE and rRMSE values also indicated only small, non-significant discrepancies between calculated and direct measurements. Small discrepancies are normal and can be attributed to minor variations when taking direct diameter measurements.

Statistical analysis showed a high degree of determination coefficient ($r^2 = 0.9728$) between the calculated and directly measured diameters. In addition, a regression coefficient (r = 0.9863) close to 1 and a free term close to 0 in the regression equation indicate strong correlation between calculated and directly measured diameter values (Figure 4). These results show a high degree of reliability of both the correlation coefficient and the parameters of the regression equation ($\alpha = 0.05$). Therefore, the calculated diameter values closely matched the directly measured diameter values, validating the methodology, which can be applied to future research efforts.

For practical purposes using the annotations in Figure 3, methods described herein can be used to accurately calculate trunk diameter at any height as well as other measurement parameters by following the steps listed next:

1. Trunk diameter at measuring equipment height level using two perpendicular direct measurements.

2. Distance BA from a measuring device to the geometric center of the trunk at measuring device height level by adding the radius AK at that point to distance BK from the device.

3. Distance to higher measuring points D along the trunk.

4. Vertical angle (α) at point B.

5. Length of line AC from the center of the cross-section of the trunk at measuring device height level to the geometric center of the ellipse (C) using the formula: $AC = AB \cdot \tan(\alpha)$.

6. Distance from the measuring device to the center of the ellipse using the formula: $BC = AB \cdot \cos(\alpha)^{-1}$.

7. One-half the major axis of the ellipse using the formula $BC - BD = CD$.

8. Accurate trunk diameter d_2 is obtained by doubling the radius using the formula:

$$d_2 = 2 \cdot OD = 2 \cdot [CD \cdot \cos(\alpha)] \, .$$

5 Conclusions

When measuring trunk diameters at heights above the measuring device level (1.58 m) and the observer is looking at an upward angle from below, the imaginary cross-section becomes slanted and appears as an ellipse. The elliptical shape of the cross-section becomes more pronounced the shorter the distance from the TC407 to the trunk, or the higher the point at which the trunk diameter is measured. The circular shape of the cross-section is only seen at dbh or at the measuring equipment height level. The above algorithm and methods described herein can be utilized to accurately determine the trunk diameter at any height on a tree. The tree cutting and diameter mensuration are often used as a reference comparison when measuring the upper diameter of the trunk. The mensuration procedure described herein will be useful as an exact method in developing new or improved techniques, such as photogrammetry and terrestrial laser scanner point cloud methods, for remote measurement of tree diameters.

Acknowledgments

We thank Bogdan M. Strimbu (Editor) and the anonymous reviewers for their thoughtful critiques that improved the manuscript.

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