A HIERARCHICAL TIMBER ALLOCATION MODEL TO ANALYZE SUSTAINABLE FOREST MANAGEMENT DECISIONS

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ABSTRACT. A two-level Hierarchical Timber Allocation model was developed that iteratively negotiates medium-term (sustainable forest management) decisions with operational (lumber production) plans. At the medium-term level, a multi-criteria timber allocation model optimally allocates forest land units, called stewardship units, to different forest products companies based on five sustainability criteria: profit, employment, wildlife habitat, recreation, and visual quality. At the operational level, a sawmilling model maximizes the profits resulting from optimally converting the timber allocated by the medium-term level into lumber products. An iterative algorithm was developed in which the decisions generated by the two hierarchical levels reach a mutually beneficial solution. The model is demonstrated in two cases and conclusions are presented about future development.

Keywords: hierarchical planning, timber allocation, multi-criteria, sustainable forest management, optimization

1 INTRODUCTION

This study addresses two difficulties that occur in sustainable forest management planning: the integration of different planning levels (operational, medium, and long term), and the multi-criteria nature of the planning problem. The first difficulty arises from the concept that organizations must secure their long-term success and survival by improving their effectiveness rather than their efficiency (Hofer and Schendel 1978). Most forest products companies are examining methods and subsequent benefits of adding value to their current products (Cohen 1992) in order to become more efficient. They should also integrate their operational decisions into the broader context of their strategic and tactical goals in order to increase their effectiveness. Therefore, forest products companies need new and improved methods of addressing current operational problems, while concurrently meeting their medium and long-term commitments, such as: maximizing net social benefits and minimizing forest ecosystem disturbance.

Sustainable forest management planning has always relied on timber allocation models to connect decisions at different planning levels (strategic, tactical, and operational). According to Colberg (1996), analysts have formulated large mathematical models with structural variables representing every conceivable resource allocation in order to serve the forest-to-product planning range. The works of Westerkamp (1978), Barros and Weintraub (1982), Hav and Dahl (1984) are just a few examples of forest-to-product timber allocation models. These models integrated various activities, such as: managing timberlands, buying/selling logs, and supplying timber to processing plants. Although valuable planning tools, the models have tried to deal with everything at once and made no distinction between the relevance of variables in addressing different levels of decision-making. In order to alleviate these situations, decision-makers usually had to either discard or aggregate variables of interest. Valuable information was therefore lost or disregarded. For example, by integrating lumber processing decisions into timber allocation models, the resulting models became so large and complex that details such as lumber and intermediary log products needed to be either aggregated into classes of products, or discarded altogether. Consequently, valuable relationships between the timber allocation and the manufacturing decisions (e.g. what product should be made from what log) were ignored. resulting in sub-optimal allocation decisions.

The second difficulty in sustainable forest management planning is dealing with multiple objectives of many stakeholders. The challenge occurs when regional social, ecological, and cultural issues rapidly become of equal or, in some cases, even more important than wood availability. Recently, timber allocation models have shifted their focus toward a multi-criteria approach. Goal programming (GP) has been used often in multi-criteria allocation models in forestry. The works of Arp and Lavigne (1982), Ludwin and Chamberlain (1989), van Kooten (1995), Bertomeu and Romero (2001) are just a few examples of how GP has been applied to sustainable forest management problems. These works dealt with multiple-use planning of forest lands, with goals related to recreation, timber harvesting, and wildlife. Other studies focused on wildlife habitat selection and on managing the biodiversity of forest lands.

A Multi-criteria Timber Allocation model was recently developed in Marinescu and Maness (2008) that was based on an integer goal programming (IGP) framework to assist with the analysis of sustainable forest management decisions. The purpose of the model was to allocate forest areas, called stewardship units, among different forest products companies, considering economic, social, and ecological allocation criteria. By including the profit among the allocation criteria, the model attempted to integrate medium-term forest management with operational decisions. Profit values were generated for each company and each stewardship unit by the FTP Analyzer[©], a sawmilling optimization model (Maness and Adams 1991). Although essential to the integration of medium-term and operational decisions, the inclusion of these profit values in the Multicriteria Timber Allocation model raised some theoretical and practical concerns.

First, the implementation of the Multi-criteria Timber Allocation model at various levels of management could be difficult and its results inaccurate, because it deals simultaneously with allocation decisions spreading over different time horizons (both medium and short term). For example, woodland managers interested in timber allocation decisions spanning larger time horizons (e.g. 5-10 years) might find it difficult to understand and accurately input all the sawmilling parameters and variables included in the model. Conversely, sawmilling managers could find it problematic to input forest management data into the model. Second, the Multi-criteria Timber Allocation model was developed under the assumption that profit values calculated for each stewardship unit and company would remain constant after the mix of stewardship units was allocated to the companies. In fact, these profit values could change depending on the timber composition of the stewardship units comprising the mix; therefore, the resulting allocations could be inaccurate. For example, the allocation of stewardship units containing large diameter stems, although very valuable when the stewardship units are allocated individually, could create bottlenecks at the sawing lines,

increasing operational costs and consequently decreasing the profit. In other words, the total profit generated from a mix of stewardship units could be smaller than the sum of the profits assumed to be generated from each of the stewardship units in the mix.

In order to alleviate these two problems, the Multicriteria Timber Allocation model would need to separate the medium-term and operational variables, while maximizing the achievement levels for all of the allocation criteria, including profit. Based on organizational theory and multilevel systems, the hierarchical planning (HP) method fits well with the goals and characteristics of the forest to product decision problems, such as timber allocation. HP is a method in which large and complex problems are disaggregated according to the management levels, the time horizons, and sometimes the geographical areas that they cover. The connectivity between the levels is essential to the proper functioning of the hierarchical decision process. The links between the levels need to insure consistency of data aggregation and disaggregation, so that the decision goals at each level are met. The resulting hierarchical allocation model is easier to implement because the sub-models at each level deal with decisions and goals that are specific to a certain time horizon and management problem. Consequently, the accuracy and the practicality of the allocation results could improve dramatically.

Many successful applications of HP in forestry have opened the door to the current research. Hof et al. (1992), Colberg (1996), Ogweno (1994), Dewhurst et al. (1997), Feunekes and Cogswell (1997), Cea and Jofre (2000) applied the HP method in forest management planning decisions at the strategic, tactical and operational levels, such as: the allocation of timber to processing activities, the allocation of output target values to district forests, the production of logs, and the selection of silvicultural treatments to address forest sustainability issues. These applications reinforced the potential of HP in developing timber allocation models able to retain an increased level of detail, while accounting for the integration of forest to product decisions.

This study presents the development of a two-level Hierarchical Timber Allocation model designed to deal with both the multiple horizon and the multi-criteria requirements of sustainable forest management decisions. The two planning levels are: a medium-term level, in which the Multi-criteria Timber Allocation model (Marinescu and Maness 2008) was modified and implemented, and an operational planning level consisting of a sawmilling optimization model (FTP Analyzer[©]). The goal of the medium-term level is to allocate the timber from stewardship units to different forest products companies so that the achievement levels of the sustainability criteria (profit, employment, visual, recreation, and wildlife) are maximized. The goal of the operational level is to maximize the profit generated by processing into lumber products the timber allocated to each company by the medium-term level. A third, long-term level was not included in the model because of theoretical and practical limitations. However, the list of stewardship units available for allocation by the medium-term level model was assumed to be generated by such a long-term level model.

The connectivity between the two levels is based on an algorithm inspired by Heal (1969) and Hof et al. (1992). In this algorithm, the medium-term level model generates optimal timber allocations that are iteratively negotiated with the operational level models. In turn, the sawmilling model communicates back to the mediumterm model the marginal values (i.e. shadow prices) of the allocated resources (i.e. stem classes). Then, the medium-term level model generates a new allocation and the iterative procedure continues until the operational level reaches optimality (i.e. maximizes the total profit). At that point, both the medium-term and the operational levels reach their goals and the optimal allocation is achieved.

This paper first presents a primer on the theory of hierarchical planning, followed by the description of the Hierarchical Timber Allocation model. Then, the model and the iterative negotiation algorithm are described and demonstrated in two scenarios: Equal Weights and Profit Max scenarios. The paper concludes with two policy analyses performed using the Hierarchical Timber Allocation model and proposes future research.

2 Methods

2.1 Hierarchical Planning According to the theory of hierarchical systems, a hierarchy is a vertical arrangement of subsystems, with priority of action or right of intervention of the higher level subsystems on the lower level ones and a dependence of the higher level subsystems on the performance of the lower level ones. Figure 1 illustrates a hierarchical system with n levels and the data flow within and between the levels.

A hierarchical system deals with two classes of data: the *input-output* and the *inter-level* connectivity data. The input-output data consist of the parameters and the variables pertaining to the part of the system described at each level. The inter-level data are comprised of the intervention data communicated from the higher levels to the lower ones, usually comprised of aggregated parameters, describing the decisions imposed by the upper levels on the lower ones. The feed-back data are usually disaggregated, detailed data (e.g. marginal productivities, utilization rates for each resource variable) describing the response of the lower levels to the deci-



Figure 1: The structure of a multi-level hierarchical system and the data used at each level (Mesarovic et al., 1970).

sions imposed by the upper-levels.

Anthony (1965) classified decisions into three categories: strategic or long-term planning, tactical or medium-term planning, and operations control. The strategic and tactical levels keep the organization moving in the right direction; i.e. they guarantee its effectiveness. The operational level is concerned with the organization's efficiency. Consequently, the variables, parameters and constraints involved in the models at different levels need to reflect the characteristics of the decisions involved at each particular level. According to Hax and Meal (1975), tactical decisions are associated with aggregate production planning (e.g. material requirements planning, aggregate production planning), while operational decisions are an outcome of the disaggregation process (e.g. daily/weekly production scheduling). However, the differentiation of levels in a hierarchical system could be based on principles other than temporal and managerial. In forestry for example, the levels of a hierarchical system could also be based on the spatial or geographical composition of the planning problem (e.g. provincial, regional, landscape-unit, forest levels).

According to Gunn (1996), the most important at-

tributes of the hierarchical planning methodology are:

1. *HP* uses separate models at each level of the hierarchy.

This provides flexibility and practicality to the decision making process. Smaller models are easier to use and maintain, while addressing a larger level of detail.

2. *HP implements a rolling planning horizon methodology.*

Only the first period decisions at each level are immediately implemented. Before implementing the decisions of later periods, decision makers need to develop an updated plan for that level. Consequently, the length of the first period in each level should be equal with the length of the whole time horizon of the subsequent level.



Figure 2: Example of the rolling horizon principle applied to a five year medium-term and a one year operational plans.

Figure 2 shows a five-year medium-term plan horizon (January '02 - January '07) and a one year operational plan horizon (January '02 - January '03). Notice that the first period of the medium-term plan is equal to the time horizon of the operational plan. At a certain point in time (in this example, June '02), if new information becomes available, the time horizons are reset, or "rolled". Consequently, the updated medium-term planning horizon will begin in June '02 and end in June '07, while the operational planning horizon will begin in June '02 and end in June '03. As a result, the hierarchical planning system will always function with the newest and most accurate information possible.

3. HP deals well with uncertainty.

Detailed decisions are made at lower levels, where

more accurate information is available, while the upper levels deal with more aggregated information. Consequently, the risk of taking the enterprise in the wrong direction is drastically reduced in the event of erroneous decisions being made at lower levels.

4. *HP follows the organizational structure of most companies.*

Each level of the hierarchical planning model is aimed at each level of management in an organization. Although this helps the implementation of HP models, it might be required that managers have restricted access to the models at different levels of the planning hierarchy.

2.2 Hierarchical Timber Allocation Model The model is comprised of two decision levels: a medium-term level and an operational level (Figure 3):



Figure 3: The structure of the Hierarchical Timber Allocation model.

At the medium-term level, the Multi-criteria Timber Allocation model (Marinescu and Maness, 2008) was implemented to allocate stewardship units to a number of forest products companies. The operational level submodels consist of sawmilling optimization models (FTP Analyzer[©]), one for each of the sawmilling facilities considered in the allocation. Connectivity through the flow of data between models at each level is essential.

There are two classes of data in this model. The first class is comprised of the *input-output data* that guarantee the functioning of the sub-models and present the intermediary and final results at each level. For example, the input data are comprised of product prices, production parameters (for the operational models) and sustainability indicator values, treatment intensities, and employment values (for the medium-term model). The output data contain the results generated by each submodel, such as: achievement levels for sustainability criteria, stewardship units (SU) and volumes allocated to each company (for the medium-term model), and lumber product volumes and profits (for the operational level models). The second class of data is the *decision data* (i.e. intervention and feed-back data). The intervention data consists of allocations of stewardship units made at the medium-term level, which are then sent to the operational level models. The operational level models run the allocations and send the medium-term level feedback data consisting of marginal values for each of the stem classes and the profit values attained by each company. The medium-term level has the right of intervention over the operational level by stopping the iteration process when all the allocation criteria are maximized.

The Medium-Term Level: The Multi-criteria Timber Allocation Model The Multi-criteria Timber Allocation Model (Figure 4) is a multi-period, integer goal programming model. The model starts with a GIS database (Maness and Farrell 2004) describing the forest area under study. The Stewardship Unit (SU) is the forest unit that is allocated by the model to either a company or to reserve.



Figure 4: Multi-criteria Timber Allocation Model (Marinescu and Maness 2008).

The objective of the model is to minimize the sum of weighted deviations of each allocation goal (or allocation criterion) from its target. The integer component of the model ensures that a Stewardship Unit is not shared between companies, due to ownership constraints.

This is a multi-period, multi-criteria timber allocation model using the following mixed integer goal programming (IGP) formulation:

$$Min \ z = \frac{w_P^-}{G_P}P^- + \frac{w_E^-}{G_E}E^- + \\ + \frac{w_V^-}{G_V}V^- + \frac{w_R^-}{G_R}R^- + \frac{w_W^-}{G_W}W^-$$
(1)

Subject to:

$$\sum_{ijkt} \left(Vol_{ijkt}^{\%} \times P_{ijkt} \right) + P^{-} = G_P \tag{2}$$

$$\sum_{ijkt} \left(Vol_{ijkt}^{\%} \times E_{ijkt} \right) + E^{-} = G_E \tag{3}$$

$$\sum_{ijkt} \left(Vol_{ijkt}^{\%} \times V_{ijkt} \right) + \sum_{j} \left(Res_j \times V_j \right) + V^- = G_V \quad (4)$$

$$\sum_{ijkt} \left(Vol_{ijkt}^{\%} \times R_{ijkt} \right) + \sum_{j} \left(Res_j \times R_j \right) + R^- = G_R$$
(5)

$$\sum_{ijkt} \left(Vol_{ijkt}^{\%} \times W_{ijkt} \right) + \sum_{j} \left(R?as_j \times W_j \right) + W^- = G_W$$
(6)

$$-TotVol_{j}^{\%} - M \times binS_{ij} + \sum_{kt} \left(Vol_{ijkt}^{\%} \right) \geq -M,$$

for each *i*, *j* (7)

$$-TotVol_{j}^{\%} - M \times binRes_{j} + Res_{j} \ge -M, \text{ for each } j$$
(8)

$$\sum_{i} (binS_{ij}) + binRes_j = 1, \text{ for each } j \tag{9}$$

$$\sum_{ikt} (Vol_{ijkt}^{\%}) + Res_j = TotVol_j^{\%}, \text{ for each } j \qquad (10)$$

$$\sum_{jk} (Vol_{ijkt}^{\%} \times Vol_{ijkt}) \le MaxV_{it}, \text{ for each } i, t \quad (11)$$

$$TotVol_j^{\%} = 100\%$$
, for each j (12)

where :
$$0 \le binS_{ij}$$
; $binRes_{ij} \le 1$, are
integer binary variables. (13)

All variables are positive.

Variables:

 P^-, E^-, V^-, R^-, W^- : Negative deviations of the profit, employment, visual, recreation, and wildlife goals from their targets. Positive deviations are welcomed, so they are not included in the model.

 $Vol_{ijkt}^{\%}$: Percent of volume of stewardship unit (SU) j allocated to company i and harvested with treatment t in period k.

 Res_j : Percent of volume of SU *j* allocated to reserve throughout the entire time horizon.

 $binS_{ij}$: Binary variable indicating if SU j was allocated to company i

 $binRes_j$: Binary variable indicating if SU j was allocated to reserve

 $TotVol_{j}^{\%}$: Percentage of volume in SU *j* allocated.

Parameters:

 G_P, G_E, G_V, G_R, G_W : Profit, employment, visual, recreation, and wildlife goal targets. The goals are calculated by running the model with one goal at a time (i.e. generating allocations that will maximize each goal, one at a time).

 $\frac{w_P^-}{G_P}, \frac{w_E^-}{G_E}, \frac{w_V^-}{G_V}, \frac{w_R^-}{G_R}, \frac{w_W^-}{G_W}$: Relative weights associated with the profit, employment, visual, recreation, and wildlife goal targets. The weights could be interpreted as the relative importance of deviating by one percentage point from the respective goals. Weights are entered in the model by the user in order to prioritize different goals and usually take values from 1 to 100.

 P_{ijkt} : Profit generated by allocating SU j to company i in period k and harvested with treatment t. Profit values are generated by the FTP Analyzer[©], by running the model for each company and SU, in each period. For partial-cuts, the profit values are reduced according to the volume intensity of the partial-cut treatment.

 E_{ijkt} : Employment generated by allocating SU j to company i in period k and harvested with treatment t. Employment values¹ are calculated depending on the company, the area, the volume, the distance to sawmill, and the slope of each SU. For partial-cuts, the employment values are reduced according to the volume intensity of the partial-cut treatment.

 V_j, R, W_j : Visual, recreation, and wildlife indicator values for SU j when allocated to reserve. These values are entered in the model from the GIS database for the area under study and are scores between 0 and 1. $V_{ijkt}, R_{ijkt}, W_{ijkt}$: Visual, recreation, and wildlife indicator values for SU j when allocated to company i in period k and harvested with treatment t. The indicator values are entered in the model by the user and are scores between 0 and 1. These values depend on the harvesting method used by each company in each SU and period. Generally, for clear-cuts, these values are zero, meaning that no visual, recreation and wildlife features could exist in stewardship units harvested with clear-cutting treatments. For partial-cuts however, these indicator values can retain part of the initial score (i.e. taken from the GIS database) depending on the volume intensity of the treatment applied by each company to each SU, in each period.

 Vol_{ijkt} : Volume of timber in SU j available for allocation to company i in period k and harvested with treatment t. These values are calculated based on the tree information in the GIS database. For partial-cuts, the volumes are reduced according to the volume intensity of the partial-cut treatment.

 $MaxV_{jt}$: Maximum volume capacity (m3) of company j in period t.

M: Large number. This number should be greater than Tot Volj% in order to force SU j to be allocated to just one company.

Objective and Constrains

The objective (1) of this model is to find an optimal allocation of stewardship units to companies that will minimize the sum of weighted negative deviations of profit, employment, recreation, visual, and wildlife goals from their targets.

Constraints (2) to (6) set the targets for each of the goals and connect the allocation variables Volijkt% to the deviational variables in the objective function.

Constraints (7) and (8), in combination with constraint (9), connect the binary variables to the stewardship units to which they refer and guarantee that one stewardship unit is allocated to either one company or to reserve.

Constraint (10) requires that the sum of volumes allocated to either sawmilling or reserve does not exceed the maximum volume available in each SU j.

Constraint (11) sets the maximum volume capacity (m3) of company i in period t.

Constraint (12) is an upper bound on the volume available for allocation in each SU. In this formulation, it forces all the volume in each SU to be allocated.

Constraint (13) sets the values of the integer variables to zero or one (binary).

 $^{^1}$ Based on employment values taken from the B.C. Ministry of Forests (2002)

Unlike linear programming models, goal programming models do not generate unique solutions, but rather nondominated solutions. Therefore, the user must evaluate a number of scenarios using different goal weights before finding a suitable solution for the multi-criteria decision problem. A commonly used procedure involves first estimating the non-dominated solution set and then identifying those scenarios that are most appropriate to the problem at hand. To construct the non-dominated set, extreme weights are applied to different goals and scenarios are generated until the non-dominated solution set is revealed (Marinescu and Maness, 2008).

The Operational Level: The FTP Analyzer[©] Model The FTP Analyzer[©] is a combined lineardynamic programming optimization model² that optimizes the sawmilling activities of each company involved in the study. The objective of the model is to find the optimum set of bucking policies, cutting patterns, and production parameters that maximizes the profit generated from manufacturing lumber products. For each sawmill, the inputs into the model are the raw materials, the cut programs, the lumber products and markets, and the plant configuration. There are three categories of raw material data: quota timber, purchased timber, and purchased log distribution. Quota timber represents the amount of timber that the government allows a company to harvest in a certain period. In the case of quota and purchased timber, the input data consist of cruise files for each stewardship unit allocated to the company. Cruise files contain measurements, such as diameter at breast height (DBH) and total height, taken from sampled trees in each stewardship unit. The raw material entered in the model can be expressed also in stem volume distribution format.

The model also requires input data about the cut programs for the conversion of stems into lumber. This data is comprised of machine speeds, productivities, and costs for each machine center, such as bucking lines, sawing lines, planers, and others. In addition, lumber products and markets need to be entered for each sawmill. Each product is defined in terms of its gross and net dimensions, the position in the log, the species, the grade, and the selling price. The plant configuration data contains the capacities of each machine center.

By running the model for each company/sawmill, the stems in each stewardship unit are optimally processed into lumber according to each company's production and market parameters. After each run, a report manager compiles the results. Reports are generated for each machine center, product, and for the overall facility. Profit values generated by this model consider a multitude of costs, including transportation costs, stumpage/purchasing costs, and production costs associated with each machine center. A valuable output of this model is the set of shadow prices for each stem class that was allocated to each sawmill.

Inter-Level Connectivity There are two types of interactions between the operational and medium-term levels: a *negotiating* interaction and an *updating* interaction. In the *negotiating* interaction, data exchanged between the operational level and medium-term level are used to evaluate how close the allocations are to achieving the operational and medium-term goals. The updating interaction occurs after the allocations are implemented, when new data become available or parameter changes take place. For example, when new stewardship units are available for harvest, or some parameters have changed, the medium-term level asks the operational level for an updated list of data (e.g. new cutting programs). According to the rolling horizon principle, the time horizons and their respective periods in each level reset (roll) to start at the current time. The model is run again and new timber allocations are generated with up-to-date information.

In the Hierarchical Timber Allocation model, the negotiating interaction between the two hierarchical planning levels was developed in an iterative fashion, according to the procedures developed by Heal (1969) and Hof et al. (1992). Because the sawmilling models (FTP Analyzer[©]) at the operational level are based on a linear programming methodology, they are able to generate shadow prices for each stem class S (i.e. all combinations of length and small end diameter classes)³. To illustrate how the shadow prices are derived from these models, a simplified version of their mathematical formulation is provided below. The full model can be found in Maness and Adams (1991).

The objective of the sawmilling model is to:

$$Max \sum_{il} Lum_Sales_{il} \times Lum_Pr_{il} - \sum_{S} Stem_{S} \times Stem_Pr_{S} - \sum Tot_Op_Costs$$
(14)

Subject to:

$$\sum_{S} \sum_{B} Stm_{B}uck_{SB} = \sum_{S} Stem_{S}$$
(15)

$$\sum_{S} \sum_{B} Stm_Buck_{SB} \times Log_Recov_{LSB} =$$

$$= \sum_{C} Log_Sawn_{LC}, \text{ for each } L$$
(16)

 $^{^2}$ The model was developed by WoodFlow Systems Corp., CityplaceVancouver, StateBC, country-regionCanada. The mathematical formulation can be found in Maness and Adams (1991).

 $^{^3}$ In this formulation, stems are obtained from the trunks of the felled trees, cut to length and ready for transportation to sawmills.

$$\sum_{L} \sum_{C} Log_Sawn_{LC} \times Lum_Recov_{ilLC} =$$

$$= \sum_{il} Lum_Sales_{il}$$
(17)

Where: $Stem_S$: Volume of stem class S that was used in lumber processing.

 Stm_Buck_{SB} : Volume of the stem class S bucked with the bucking pattern B.

 Log_Recov_{LSB} : Recovery factor for the stem class S when bucked with the pattern B and converted to log class L.

 Log_Sawn_{LC} : Volume of the logs class L sawn with the sawing policy C.

 Lum_Recov_{ilLC} : Recovery factor for the lumber with dimensions i and l, sawn from a log of class L, using sawing policy C.

 Lum_Sales_{il} : Volume of lumber of dimension i and l sold.

Stem_ Pr_S : Stem price (e.g. stumpage, purchasing price) of a unit volume of stem class S.

 Lum_Pr_{il} : Sale price of a unit volume of lumber of dimension i and l

Tot_Op_Costs: Aggregate operating costs for activities such as: bucking, sawing, finishing, packaging, etc.

The objective (14) of the saw milling model is to maximize the revenue generated by the sales of lumber products, minus the costs of raw materials and operations. Constraints (15) to (17) deal with the conversion of stems of different classes S into lumber products of dimension i and l. If the dual formulation of the model presented in equations (14)-(17) was constructed, at optimum, shadow values would be available for each stem of class S, each log of class L, and each lumber product of dimension i and l. In practice, these shadow values are generated by the LP solver used in the FTP Analyzer[©]

The shadow values for constraint (15) indicate the relative value of stem classes to each sawmill. Since each stewardship unit (SU) contains a unique stem class distribution, its value to each sawmill (company) depends on how much volume of each desirable stem class exists in the stewardship unit. In order to assess how valuable each SU is to each company, a *Shadow Composite Value* is calculated by the medium-term level model for each SU and company, as follows:

$$CV_B_i_S_k = \sum_{S} Vol_Stem_{Si} \times y_{Sk},$$

for each SU *i* and company *k* (18)

Where: $CV_B_i_S_k$: Shadow composite value for SU *i* when calculated with the shadow values from the FTP

Analyzer[©] model for Sawmill k. There are $k \ge i$ such composite values.

 Vol_Stem_{Si} : Volume of stem class S in SU i.

 y_{Sk} : Marginal value (shadow price) for stem class S generated by the FTP Analyzer[©] for company k.

The search for an optimal timber allocation can be achieved through an iterative process, in which the medium-term level model finds an allocation of stewardship units that maximizes the sum of profits achieved at the operational level. According to the duality condition, at optimum, each of the sawmilling models achieves a profit equal to:

$$P_k = \sum_{S} Stem_{Sk} \times y_{Sk}, \text{ for each company } k \qquad (19)$$

By combining expressions (18) and (19), the optimal timber allocation is achieved when the medium-term model maximizes the sum of the SU Shadow Composite Values, as follows:

$$Max \sum_{k} \sum_{i} \sum_{S} (Vol_{StemSi} \times y_{Sk}) \Leftrightarrow \\ \Leftrightarrow Max \sum_{k} \sum_{i} CV_B_{i}_S_{k}$$

$$(20)$$

Concomitantly, according to the procedure devised by Heal (1969), the medium-term model proportionally increases the allocation of those stem classes with shadow prices above the average and decreases the allocation of those with values below the average. To achieve this increase, the shadow prices of each stem class are adjusted proportionally to their departure from the average before the medium-term model is run in each iteration. The adjustment is achieved using an expression derived from Hof et al. (1992). The SU composite values are then recalculated and included in the objective function (20). Consequently, by running the medium-term model with these adjusted SU composite values instead of profit values, the allocation of those stewardship units that contain the most valuable stem classes guarantees monotonically increasing profit values in each iteration.

Figure 5 presents the algorithm of the iterative negotiating process between the medium-term and the operational level models according to the procedure described in Heal (1969). The iteration process starts with an initialization stewardship unit allocated to each company. This initialization SU is fictional and is comprised of all the stem classes present in all available stewardship units. The volume of the initialization SU is equal to the maximum volume capacity of each company. Consequently, the initialization SU meets the initial allocation constraints presented in Heal (1969) and guarantees that, if the initial allocation is feasible, all the subsequent allocation solutions are also feasible. After



Figure 5: The inter-level negotiating algorithm between the medium-term and operational level sub-models in the Hierarchical Timber Allocation model.

running the operational level models with the initial allocation, the shadow values are generated for all stem classes, in each company. Figure 5 also shows that the profit values attained at each company are stored and that the initialization SU is dropped after the first iteration. The shadow values are then sent to the mediumterm level model where they are adjusted proportionally with the distance from their averages according to the following expression:

$$y_{Sk}^{*} = y_{Sk} + \frac{y_{Sk} - \overline{y}_{Sk}^{K_S}}{\sum_k y_{Sk}},$$
 (21)

for each stem class S and company k

where:

 y_{Sk}^* : New, <u>adjusted</u>, shadow price for the stem class S allocated to company k.

 y_{Sk} : Shadow price for stem class S allocated to company k in the previous iteration.

 $\overline{y}_{Sk}^{K_S} {:}$ Average of the shadow prices for stem class S calculated based on the set:

$$K_{S} = \left\{k : Stem_{Sk} > 0, \text{ or } Stem_{Sk} = 0 \text{ but } y_{Sk} > \overline{y}_{Sk}^{K_{S}}\right\},\$$

where: StemSk is the volume of stem class S allocated to company k.

The purpose of defining the set Ks was that the adjustments needed also to be applied to those facilities where the allocations of stem class S were zero, but their shadow prices were above the average from the previous iteration.

With these adjusted shadow prices, the composite stewardship unit values are calculated for each SU and each company. Consequently, stewardship units containing desirable stem classes (with shadow prices above the average) will increase their composite values, whereas the others will decrease them. The Multi-criteria Timber Allocation model then generates a new allocation of stewardship units, which are sent to the operational level models. The iterative negotiating process is stopped by the medium-term model when the total profit value achieved at the operational level is equal to that of the previous iteration. According to the procedure devised by Heal (1969), this indicates that an optimum allocation was found and that the goals of both medium-term level (i.e. maximizing sustainability criteria) and operational level (i.e. maximizing total profit) are attained.

Please note that, although only the first period allocation of stewardship units is sent to the operational level, each iteration includes ALL the stewardship units (in all periods). Consequently, the feedback data (i.e. shadow prices) from the operational level impact the mediumterm level solutions throughout its whole time horizon. 2.3 Model Demonstration The allocation procedure used in the Hierarchical Timber Allocation model was demonstrated using the same case presented in Marinescu and Maness (2008). The study area was located in the Kootenay Columbia Region of British Columbia, and consisted of two landscape units. In this area, a set of 463 stewardship units (SUs) were allocated by the Multi-criteria Timber Allocation model to three hypothetical forest products companies or to reserve. These companies operated very different sawmills: a stud mill, a dimension mill, and a diversified mill (Table 1). In this context, it was imperative for the three companies to be allocated timber that best matched their production and product specifications.

The sustainability allocation criteria used in this case were: profit, employment, visual quality, recreation, and wildlife habitat. The profit values entered in the Multicriteria Timber Allocation model were generated by the FTP Analyzer[©] for each SU and company. The allocation parameters entered in the Hierarchical Timber Allocation model remained unchanged, except the profit values, which were substituted with SU shadow composite values, according to the iterative procedure presented previously. The time horizon chosen for the case analysis was 5 years, consisting of two time periods: one year, followed by four years. A length of one year for the first period was assumed appropriate in the context of implementing a five-year plan. The profit values were discounted by 5% in the first period to adjust for the time preference of money.

As with the Multi-criteria timber allocation model, two scenarios were analyzed with the Hierarchical Timber Allocation model: the Equal Weights and the Profit MAX scenarios. The Equal Weights scenario consisted of an allocation that balanced the allocation goals (i.e. all goal weights were set to 1); whereas the Profit MAX scenario presented an allocation that emphasized the profit goal (i.e. profit weight was set to 100 and all the others to 1).

Before demonstrating the allocation procedure in the Hierarchical Timber Allocation model, the lists of stewardship units allocated to each of the three companies by the Multi-criteria Timber Allocation model were fed into the FTP Analyzer[©]. This operation was performed in order to examine whether the resulting profit values were consistent with those guaranteed by the Multi-criteria Timber Allocation model. Any differences would indicate the magnitude of profit loss that could occur if the allocation generated by the Multi-criteria Timber Allocation model was implemented. These differences would also confirm the need for the iterative negotiating process devised in the Hierarchical Timber Allocation model.

An analysis of the iterative negotiating process was

		Company 1	Company 2	Company 3
Product Type		SPF Boards and	SPF Studs	SPF Dimension Lumber
		Dimension Lumber		(Japanese grades)
Markets		Canada, US	Canada, US	Canada, US, Japan
Capacity $(1,000 \text{ m}^3)$	Period 1	350	200	250
	Period 2	1400	800	1000
Employment (avg. persons/yr.)		70	23	46

Table 1: Production and market parameters for Companies 1, 2, and 3.

also performed in order to showcase the capability of the Hierarchical Timber Allocation model to find those timber allocations that matched the production requirements of the three companies, consequently increasing the total profit. In this analysis, only the results of the Profit MAX scenario were analyzed because they required more iterations than those of the Equal Weights scenario. Consequently, analyzing the convergence toward the optimal solution provides a better understanding of the allocation process. After each iteration, the allocation results were stored and later utilized to graph how the model converged toward the optimal allocation decision.

3 Results and Discussions

One of the concerns with the Multi-criteria Timber Allocation model was that it could not guarantee the profit values generated by its static allocation procedure. The results of the Multi-criteria Timber Allocation model showed that, had this allocation been implemented in practice, a lower total profit vale would have been obtained (Figure 6, left). However, the results obtained with the Hierarchical Timber Allocation model were not only accurate (Figure 6, right), but also larger than those obtained by the Multi-criteria Timber Allocation model for the same data set.

The iterative procedure designed in the Hierarchical Timber Allocation model to negotiate the timber allocations between the medium-term and the operational level models produced a series of intermediate results. These were analyzed in order to understand and demonstrate the convergence toward the optimal solution.

In the Equal Weights scenario, the optimal solution was achieved in one iteration. Figure 7 shows the profit values obtained at the operational level by each of the operational models and in total. The graph suggests that, in Iteration 0 (i.e. model initialization), the allocation of the initialization stewardship units produced different profit values for each company. The reasons for this are: a) the companies had different product structures and prices, b) the incompatibility between the tim-



Figure 6: The comparison between the profit results generated by the Multi-criteria Timber Allocation and the Hierarchical Timber Allocation models (Profit MAX and Equal Weights scenarios).

ber composition of the initialization SUs and the production parameters of each company, and c) the maximum volume capacity of each company. In Iteration 1, the profit values for all three companies increased considerably due to the shadow price adjustments that were applied after Iteration 0. An interesting case occurred with Company 3, which did not respond well to the initial allocation. In iteration 1, however, this company achieved a profit value higher than those of Companies 1 and 2. In Iteration 2, the total profit value was identical with that of Iteration 1; therefore the procedure was stopped by the medium-term level model.

In contrast to the Equal Weights scenario, the Profit





Figure 7: The convergence of the profit values of each company and in total toward the optimal solution in the Equal Weights scenario.

MAX scenario achieved optimality in two iterations. Figure 8 shows that, because the initial iteration was identical in both allocation scenarios, the same profit values were achieved as in the Equal Weights scenario. In Iteration 1, however, all profit values were higher than those achieved in the Equal Weights scenario. This was expected given that the Profit MAX scenario emphasized the profit goal. Unlike the Equal Weights scenario, Iteration 2 did not produce the same allocation as in the Iteration 1. In this iteration, Companies 1 and 2 increased their profit values, which indicated that they were allocated a different mix of stewardship units than in the previous iteration. Company 3, however, did not change its profit value, which suggested that it received the same mix of stewardship units as in previous iteration. In Iteration 3, no change occurred in the total profit values, therefore the procedure was stopped.

The above results indicate that the iterative procedure produced increasing profit values, converging toward the maximum. To analyze and demonstrate even further the iterative allocation procedure, the stewardship units allocated to each company in the Profit MAX scenario were converted into stem class distributions. By plotting these distributions, one is able to visualize how, after

Figure 8: The convergence of the profit values of each company and in total toward the optimal solution in the Profit MAX scenario.

each iteration, the Hierarchical Timber Allocation model was able to fit the raw material requirements of each company with the timber composition of the allocated stewardship units.

Figure 9 presents the stem class distributions of stewardship units allocated to Company 1 in Iteration 0 (plotted as hatched bars) to Iteration 3. These distributions are presented as percentages of the total volume of timber allocated. The stem classes are combinations of small end diameter (SED) (4-9 in., 10-15 in., 16-21in., and 22-27 in.) and length (8-20 ft., 21-33 ft., 34-46 ft., and 47-60 ft.) classes. Since the initialization SU was allocated to each of the three companies in Iteration 0, the stem distribution in this iteration was identical for all the companies. Figure 5.8 suggests that the initialization SU contained a large volume (>65%) of the small SED (4-9 in.), large length (34-60 ft.) stem classes. The most dramatic changes in volumes occurred in Iteration 1, where the product structure of Company 1 (i.e. most lumber products had widths between 4 and 12 in.) required the model to increase the volumes of 10-15 in. SED / 34-46 ft. length class, while drastically decreasing the volume of stem in 4-9 in. SED / 47-60 ft. length class.



Figure 9: The stem class distributions of timber allocated to Company 1 in Iteration 0 to 3 in the Profit MAX scenario.

These adjustments were the result of the allocation model shifting the allocation of stewardship units toward the ones that could produce the most valuable lumber products (2x10 and 2x12). The distributions presented in Iterations 2 and 3 confirm this presumption by further adjusting the volumes of the two stem classes. Note that in Iteration 2, the model needed to adjust the volumes of the stem classes allocated in the previous iteration, which indicates that Company 1 contributed significantly to the need for additional iterations.



Figure 10: The stem class distributions of timber allocated to Company 2 in Iteration 0 to 3 in the Profit MAX scenario.

Figure 10 presents the volume distributions of stem classes allocated to Company 2, in Iterations 0 to 3. The graph suggests that, in Iteration 1, the allocation model increased the volumes of stems in the 4-9 in. SED / 34-46 ft. length class and 10-15 in. SED / 34-46 ft. length class. However, the model decreased the volumes of 10-15 in. SED / 47-60 ft. length class. This action was justified by the need of Company 2 to produce studs with widths of 4 and 6 inches and lengths of 8 and 9 feet. The next two iterations did not produce significantly large adjustments in the allocated stem class distributions, an indication that the model reached the desired timber distribution for Company 2 faster than Company 1.



Figure 11: The stem class distributions of timber allocated to Company 3 in Iteration 0 to 3 in the Profit MAX scenario.

Figure 11 presents the distribution of stem classes allocated to Company 3 in Iterations 0 to 3. At first glance, the graph indicates that Iterations 1, 2 and 3 produced the same allocations, which was expected since the profit values generated by Company 1 in these iterations were identical. The fact that the allocation model quickly found an optimal solution could be partially explained by the distinct set of products that Company 3 produced (e.g. vertical grain Japanese grades - Hirakaku), for which there were no other similar competitors. In turn, the sawmilling parameters (sawing and bucking patterns) required that the timber allocated to Company 3 contain larger SED classes than those required by Companies 1 and 2. The distribution of stem classes presented in Figure 5.10 proves this supposition: in Iterations 1, 2, and 3, the model drastically reduced the volumes of stems in the 4-9 in. SED classes and substantially increased the volumes of stems in 10-15 in. and 16-21 in. SED classes.



Figure 12: Allocation map in the Wildlife Corridor policy case (Equal Weights Scenario)

The above analysis demonstrates how the Hierarchical Timber Allocation model was able to achieve increasing profit values at the operational level by allocating the right raw materials to the right sawmill. The results indicate how the iterative procedure shifted the composition of raw materials from an inappropriate distribution of stem classes (initialization SU) to those distributions required by each company. These outcomes demonstrated the iterative negotiating procedure and showed how the Hierarchical Timber Allocation model could produce optimal multi-criteria timber allocations that satisfy all the conditions of sustainable forest management.

3.1 Policy analyses The Hierarchical Timber Allocation model was also demonstrated in two policy analyses, which imposed constraints on the model in different ways. To model the sustainable forest management planning conditions, only the results of the Equal Weights

scenario were analyzed and compared with those of the unconstrained model.

The Wildlife Corridor Policy Connectors between core habitat areas, called *wildlife corridors*, allow wildlife to migrate, mate, and feed. Wildlife corridors must be large enough to allow for easy movement of the largest species and be carefully managed. This is especially important for migratory animals, large predators, and those with large home ranges (such as ungulates).

The Wildlife Corridor policy imposed constraints on the model regarding the implementation of a wildlife corridor in the study area, which reduced the availability of timber and the choice of harvesting treatments. The GIS database (Maness and Farrell 2004) indicated that a wildlife corridor comprised of 34 stewardship units existed on the Eastern border of the study area (Figure 12). Consequently, clear cuts were banned and only low intensity partial cuts were allowed in the stewardship units comprising the corridor.

The trade off analysis (Figure 13) indicates the changes in criteria values that could occur if this policy was implemented. It also increased knowledge about the forest values in the wildlife corridor. Note that, besides a significant wildlife habitat component (i.e. a 13% increase in wildlife objective), there were other values that benefited from this policy, such as: visual and recreation (1% and 3% increase, respectively). The profit and employment indicators suffered little or no changes, indicating that the wildlife corridor constraints did not impose a stricter restriction on harvesting than in the unconstrained model.



Figure 13: The impact of the Wildlife Corridor policy on criteria in the Equal Weights scenario.

The Accessibility Policy Timber accessibility has always had a severe impact on timber availability and, therefore, has constituted an important constraint on timber allocation activities. Generally, due to scarce networks of forest roads, large forested areas are considered inoperable. Roads are usually very costly to build and maintain, so many forest products companies prefer to harvest in the stewardship units that contain valuable timber and are located as close as possible to existing roads. As a result, the distance to roads and the subsequent costs can be deciding factors in timber allocation activities.

The Accessibility Policy required that the stewardship units allocated for harvesting to companies were located as close as possible to existing roads. This policy could be beneficial to forest ecosystems, because it could create large, continuous reserve areas located further away from human activities. However, they could dramatically restrict access to valuable timber. In the two landscape units under investigation, the GIS database indicated two forest roads (Figure 14): one on the southern part (positioned east - west) and the other on the eastern part (positioned north - south). From the total of 463 stewardship units, 164 have access to these roads (i.e. at least one road traverses the SU). In order to model the Accessibility Policy, the Hierarchical Timber Allocation model minimized the sum of distances between the stewardship units and the closest roads in addition to minimizing the deviations from the goal targets.



Figure 15: The impact of the Accessibility policy on criteria in the Equal Weights scenario.

The trade-off analysis (Figure 15) shows that large decreases in profits and employment values could occur as a result of this policy scenario, but also loss of recreation potential and wildlife habitat. This indicated that valuable timber was located further away from these roads and new roads would need to be built to access it. Company 2 (the stud mill) suffered the most as a result of this policy as very little timber was allocated to it. Consequently, Company 2 incurred a large drop in profit, mostly because of its low value products and lack of product diversity. The other two companies, incurred smaller profit losses, but they were allocated more timber than Company 2, because of their high value products and higher product diversity.

The results demonstrate that the Hierarchical Timber Allocation model seem to address closely the sustainable forest management conditions. In addition to the strengths inherited from the Multi-criteria Timber Allocation model, the dynamic allocation procedure enhanced the accuracy of the allocation solutions.

Legend



Figure 14: Allocation map in the Accessibility policy case (Equal Weights Scenario).

4 Conclusions

This study presented the development of a Hierarchical Timber Allocation model capable of iteratively finding optimum timber allocations that met the goals of both the medium-term plan (i.e. maximization of forest sustainability indicators) and of the operational plan (i.e. maximization of profits in each company). By incorporating the Multi-criteria Timber Allocation model at the medium-term level and connecting it with the sawmilling models at the operational level, the objectives of the multi-criteria allocation and the forest to product integration were met.

Designing a dynamic connectivity between the medium-term and operational level was essential, as it guaranteed that the timber allocation produced by the medium-term level generated monotonically increasing profit values at the operational level. Consequently, when the optimum solution was reached, the goals of the two levels were met and the dynamic link between the forest and the end product was achieved. The need for a dynamic connectivity was demonstrated in a case analysis where the profit results of the Multi-criteria Timber Allocation model were compared against those of the Hierarchical Timber allocation model. The latter model produced more profitable results because the allocation at the medium-term level was guided iteratively by the operational level models. Therefore, the profit values obtained with this model were guaranteed to reflect the most current operational parameters. In addition, the intermediate allocation results generated after each iteration were presented. These results not only validated the convergence of the profit values toward the optimum solution, but also the capability of the model to match the lumber production requirements of each sawmill with the composition of the timber allocated to them.

In order to further validate the Hierarchical Timber Allocation model and showcase its capabilities, two policies were modeled and analyzed: the Wildlife Corridor and the Accessibility policies. The results, produced in both numerical and graphical format, validated the model and presented the relationships between the forest management and the lumber manufacturing operations. Through a series of trade-off analyses users of the model can gain a better understanding of the costs associated with implementing them. In conclusion, because of its capability to integrate forest to product decisions and its flexibility, the Hierarchical Timber Allocation model could benefit analysts to better understand the complex relationships between different sustainable forest management activities.

The Hierarchical Timber Allocation model could benefit from the implementation of a third, strategic level, which could address issues of long-term sustainability of the forest ecosystem. It would also provide an accurate list of Stewardship Units to the medium-term level for allocation. Also, including a stakeholder input methodology would result in a more meaningful prioritization of sustainability indicators and subsequent scenario analyses. From a purely scientific point of view, the model could be applied to better understand what the definition of Sustainable Forest Management is and to further validate sustainability Criteria and Indicators.

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