MODELING CARIBBEAN TREE STEM DIAMETERS FROM TREE HEIGHT AND CROWN WIDTH MEASUREMENTS

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Abstract. Regression models to predict diameter at breast height (DBH) as a function of tree height and maximum crown radius were developed for Caribbean forests based on data collected by the U.S. Forest Service in the Commonwealth of Puerto Rico and Territory of the U.S. Virgin Islands. The model predicting DBH from tree height fit reasonably well (R2 = 0.7110), with strongest in subtropical moist and wet forest. The model predicting DBH from crown radius fit the data poorly (R2 = 0.2876), but improvements were made when the model was fit by forest life zone and crown radius measurement protocol. Models fit with both maximum crown radius and tree height had R-square values that ranged from 0.1803 for the subtropical dry forest to 0.8018 for the subtropical moist forest life zone where crown radius was measured with urban forest inventory protocols. Tree heights had stronger correlations with DBH than did crown radius, perhaps due to difficulties in measuring tree crown width or natural variability in this hurricane-disturbed environment. Models that use tree height have some potential for predicting DBH for use in Caribbean forest biomass and carbon estimation models, but the potential for error propagation by using DBH predicted from crown radius is too great to earn our recommendation for such applications.

Keywords: Allometric models; secondary forest; Puerto Rico; U.S. Virgin Islands

1 Introduction

Understanding regional and global forest biogeochemical cycles so that informed decisions can be made regarding their management requires accurate estimates of forest structure, biomass and carbon over landscape or larger scales. Direct measurements of forest structure are taken on intensively sampled, relatively small field plots, and these data are used to create allometric models that predict forest parameters like volume, biomass and carbon from easily measured tree attributes. This allows for the expansion of these estimates over greater expanses of forest. Diameter at breast height (DBH) is commonly used as a predictor of other tree metrics in a wide variety of allometric equations. Numerous tree biomass equations use DBH as a predictor variable, with notable examples developed for subtropical and tropical forests [1, 2].

Installation of enough field sampling plots to obtain adequate numbers of DBH measurements is sometimes too costly or difficult in rough terrain or areas that are difficult to access on the ground (e.g. periodic flooding, dense vegetation, etc.), conditions often found in the humid tropics. Estimating DBH from tree metrics that can be measured remotely facilitates landscape and regional scale biomass and carbon estimation. In an early example of this approach, Perez [3] modeled DBH from crown widths measured on aerial photographs in Puerto Rico, Dominica and Thailand. More recent efforts have focused on measuring individual tree heights using lidar data [4-6] or crown widths from high resolution aerial [7] and satellite imagery [8, 9], then using the modeled DBH to estimate total-tree biomass and carbon.

The objective of this study was to develop models to predict tree DBH from tree height and crown radius measurements for Caribbean forest Holdridge life zones [sensu 10] (subtropical dry, subtropical moist, subtropical wet/rain and lower montane) and mangrove forests. The goal was to find models that use variables derived from remotely-sensed data and that would be suitable for estimating tree metrics needed to calculate forest biomass and carbon.

2 Methods

2.1 Study area and forest inventories The tree measurements came from two sources: U.S. Forest Service Forest Inventory and Analysis (FIA) forest inventory plots measured in 1980, 1990, and from 2001 to 2004 on the islands of Puerto Rico, Vieques and Culebra in the Commonwealth of Puerto Rico, and on the islands of St. Croix, St. John, and St. Thomas in the Territory of the U.S. Virgin Islands; and U.S. Forest Service Urban Forest Effects (UFORE) inventory plots measured in 2002 in the San Juan Bay Estuary watershed in San Juan, Puerto Rico. The trees measured in FIA plots were in closed canopy stands while those measured on UFORE plots ranged from closed-canopy forest patches to open-grown street and yard trees.

Tree DBH was measured at 1.4 m for all trees with DBH $\geq$ 2.5 cm on both FIA and UFORE plots. On all plots, total tree height (H_T) measurements were taken to the top of the live crown on all live trees with DBH $\geq$ 2.5 cm using a combination of clinometers, Hagloff Vertex III hypsometers, and measurement tapes. Two different protocols, however, were used to measure crown width. On the FIA plots, crown width was recorded to the nearest one-tenth meter by two measurements: longest radius (R_{LONG}) from the bole to drip line and shortest radius (R_{SHORT}) from the bole to drip line, for each live tree with DBH $\geq$ 12.5 cm [for additional tree measurement details see 11]. Crown width on UFORE plots was recorded to the nearest one-tenth meter on trees with DBH $\geq$ 12.5 cm by two measurements: North-South (D_1) and East-West (D_2) widths, drip line to drip line, along the bole [for additional tree measurement details see 12]. In order to make the two datasets as compatible as possible for combined modeling, maximum crown radius (R_{MAX}) was calculated for each set of trees. For the trees measured on the FIA plots, $R_{MAX} = R_{LONG}$. For the trees measured on the UFORE plots,

$$R_{MAX} = \max(D_1/2, D_2/2) \tag{1}$$

Calculation of R_{MAX} for the UFORE trees assumes that the midpoints of the crown diameters intersect the tree bole. A test of hypothesis H_0: $D_1 = D_2$ was not rejected (p-value = 0.4014) indicating no directional bias, that is, the North-South crown widths were not longer or shorter on average than the East-West widths.

2.2 Model fitting A linear model form was selected for modeling DBH from the predictor variables H_T and R_{MAX}:

DBH = $b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n$ (2)

We fit models with H_T and R_{MAX} separately, as well as models with both predictor variables together. Additionally, we fit these models by Holdridge life zone to further refine the models with ancillary information that would be commonly available. We used plot center coordinates to extract the Holdridge life zone of each plot from a digitized version of the map that appears in Ewel and Whitmore [10].

Tree DBH and height data were taken from forest inventories conducted in 1980, 1990, and from 2001 to 2004. Only the first measurement of trees that had been measured repeatedly was kept in the dataset. Trees in the Caribbean frequently experience crown and stem damage from hurricanes, and hurricanes Georges (1989) and Hugo (1998) severely damaged forests in Puerto Rico and the U.S. Virgin Islands during the data collection period. We chose to remove trees with damaged stems, tops, or branches noted by the field crew, as opposed to retaining these trees as done in Kenefic and Nyland [13]. Additionally, mean height to diameter ratios were calculated for each tree species. Trees with height to diameter ratios exceeding the mean plus 2 standard deviations were flagged as potential outliers. After further examination of potential outliers in scatter plots, a total of 965 trees were excluded from the data set used to model DBH from H_T.

After the initial model fitting, scatter plots of the residuals were generated. Distribution of the residuals indicated the possible need for a natural log transformation of both H_T and DBH [pages 541-544 in 14]. Since it is well known that the logarithmic transformation results in biased estimates, both transformed and untransformed models were fit to the data. The SAS procedure REG was used to fit the final model of form:

$$\ln(DBH) = b_0 + b_1 \ln(H_T) \tag{2}$$

or equivalently,

$$DBH = e^{b_0} * H_T^{b_1} \tag{3}$$

To fit models that predict DBH from crown widths, trees from both the FIA and UFORE plots were included, but the data set was limited to trees most visible in overhead images, that is, trees in the open-grown, dominant, and co-dominant crown classes. After the initial model fitting, scatter plots of the residuals were generated. From these plots, thirteen observations were identified as outliers and subsequently removed from the dataset before the final models were fitted. Note that crown width measurements were made on only a subset of forest inventory plots measured in 2001 to 2004, so this data set is much smaller than the data set used to model DBH from H_T. As previously described, the scatter plots of the residuals resulting from initial model fitting indicated the need for a natural log transformation of both R_{MAX} and DBH. The SAS procedure REG was used to fit the final model of form:

$$\ln(DBH) = b_0 + b_1 \ln(R_{MAX}) \tag{4}$$

or equivalently,

$$DBH = e^{b_0} * R_{MAX}^{b_1} \qquad (5)$$

We then fit models with both H_T and R_{MAX} as predictor variables. This data set was slightly reduced due to missing tree heights for some trees with crown width measurements.

$$\ln(DBH) = b_0 + b_1 \ln(H_T) + b_2 \ln(R_{MAX}) \qquad (6)$$

or equivalently,

$$DBH = e^{b_0} * H_T^{b_1} * R_{MAX}^{b_2} \qquad (7)$$

3 RESULTS

A total of 13,764 tree measurements taken on FIA plots in all forested life zones found on the islands were used for modeling DBH with H_T (table 1) and 2,739 tree measurements (2,552 forest trees across all life zones and 363 urban inventory trees from the subtropical moist forest life zone only) were used for modeling DBH with R_{MAX} (table 2).

3.1 Models to predict DBH from tree height and crown radius All models predicting DBH from H_T (table 3), R_{MAX} (table 4), and H_T with R_{MAX} (table 5) overall and by Holdridge forest life zone were significant at the 0.05 alpha level. Variation explained by the model with H_T as the sole predictor variable exceeded 71% ($R^2 = 0.7110$), and was highest for subtropical wet forest ($R^2 = 0.7263$) and lowest for lower montane forests ($R^2 = 0.3643$) (table 3).

Models with R_{MAX} as the sole predictor variable explained less variation in DBH (table 4). Variation explained by the model was highest for the lower montane life zone ($R^2 = 0.4398$ for models with R_{MAX} alone and 0.4226 for models with R_{MAX} and H_T) and lowest for the subtropical dry forest ($R^2 = 0.1575$ and 0.1803). Results indicated that improvements to the subtropical moist forest model, the only life zone with both FIA and UFORE plots, might be possible if fit by crown width measurement protocol. Indeed this was the case as R^2 for the subtropical moist forest UFORE trees improved to 0.7741 from 0.1466 (table 4). The addition of H_T to the R_{MAX} models, however, had little effect on their predictive ability (table 5).

The untransformed model 5b was fit to the data used for model 5a fits. Results (table 6) indicated that all parameter estimates were significantly different from zero with the exception of the height exponent (b_1) for the Mangrove life zone and the FIA protocol. That equation was refit with the height exponent set to zero. Maximum crown radius (R_{MAX}) was replaced with crown

area in an attempt to improve model fits, but the resulting fit statistics did not indicate consistent improvement over using R_{MAX}. Total tree height (H_T) was replaced with height above DBH (H_T-1.37) in an attempt to improve estimates of trees just above DBH but again improvement in fit statistics did not warrant modifying the model.

4 DISCUSSION

Many studies have explored the relationship between H_T and DBH [thoroughly reviewed in 15, 16, 17] with the objective of predicting the harder to measure H_T metric from the more easily obtained DBH measurement. Although much of this work has focused on coniferous species, temperate and tropical broadleaf trees also have shown strong H_T and DBH correlations [13, 18, 19] despite their more variable branching patterns and growth forms.

Results of these previous studies show that our models predicting DBH from H_T for Caribbean trees growing in the subtropical moist and subtropical wet forest life zones are of slightly poorer fit than the norm in temperate and tropical hardwood forests, but they still could be used with an understanding of their limitations. Our models for subtropical dry, subtropical lower montane, and mangrove forests, however, are of marginal utility. Variation in DBH explained by H_T was lowest in the subtropical lower montane forest life zone and mangrove forest type. Tree sample size was substantially reduced in these areas. The systematic forest inventory placed few plots in the small, high elevation montane forests and narrow, coastal bands of mangrove forest. Also, the variety of forest types and growth forms within the montane forests, ranging from palms forests to elfin woodlands, complicated fitting of a single model for that life zone.

Although Palace et al. [9] presented an equation to estimate DBH from crown width for tropical forests in the Amazon region (R^2 value of 0.57), it is more common to see studies that present models estimating crown diameter from DBH measurements. Studies show that tree DBH is the best predictor of crown width for both broadleaf and coniferous trees in the continental United States [20-22] and tropical forests in the New World and Old World [3, 8, 9, 23]. The model fits in this study of Caribbean forests, however, generally were poorer than those found in other comparable studies, and the addition of H_T to the models produced only minor improvements in predictive ability. Weaver and Poole [23] fit allometric equations to the relationship between crown diameter and DBH for four species in the Puerto Rican Commonwealth forests subtropical dry (Guánica), subtropical moist (Cambalache), and subtropical wet (Mar-

Table 1: Ranges of the data used to fit Equation 3 by Holdridge life zone.

Life zone	N	DBH				Height, total		
		Mean	Max.	Min.		Mean	Max.	Min.
		- - - - - - cm - - - - - -				- - - - - - m - - - - - -		
All	13764	12.5	108.6	2.5		8.3	40.0	1.5
Subtropical dry forest	1133	7.4	66.8	2.5		5.5	19.0	1.7
Subtropical moist forest	8829	11.9	105.0	2.5		8.1	37.0	1.5
Subtropical wet forest	3428	15.2	37.0	2.5		9.7	40.0	1.5
Lower montane	286	15.7	69.4	2.5		8.1	22.2	2.0
Mangrove	88	14.9	33.0	2.5		9.8	17.0	2.4

Table 2: Ranges of the data used to fit Equations 4 and 5 by Holdridge life zone and measurement protocol.

Life zone	Protocol	N	DBH				Height, total				Max. radius		
			Mean	Max.	Min.		Mean	Max.	Min.		Mean	Max.	Min.
			- - - cm - - -				- - - m - - -				- - - m - - -		
All	All protocols	2739	21.1	60.0	2.5		12.1	35.1	1.5		3.7	11.1	0.2
	FIA	2552	21.4	60.0	12.5		12.4	35.1	1.5		3.8	11.1	0.2
	UFORE	187	16.8	50.5	2.5		8.6	25.5	1.9		2.6	8.3	0.4
Subtropical moist forest	All protocols	1585	20.8	60.0	2.5		12.0	31.5	1.5		3.7	11.1	0.2
	FIA	1398	19.5	54.7	12.5		8.7	19.0	1.5		3.7	8.6	1.0
	UFORE	187	16.8	50.5	2.5		8.6	25.5	1.9		2.6	8.3	0.4
Lower montane	FIA	115	21.1	57.4	12.5		11.3	22.2	3.0		3.3	8.5	1.0
Subtropical dry forest	FIA	225	19.5	54.7	12.5		8.7	19.0	1.5		3.7	8.6	1.0
Subtropical wet forest	FIA	735	22.3	59.3	12.5		13.6	35.1	2.0		3.9	9.2	0.2
Mangrove	FIA	79	18.9	33.0	12.6		11.4	17.0	4.0		3.2	6.0	1.0

Table 3: Model statistics and parameter estimates from DBH prediction Equation 3a by Holdridge life zone.

		$\ln(DBH) = b_0 + b_1 \ln(H_T)$					
		Model Statistics			Parameter Estimates		
Life zone	N	r^2	RMSE	Pr>F	b_0	b_1	
All	13764	0.7110	0.4629	< 0.001	-0.2769	1.2522	
Subtropical dry forest	1133	0.5226	0.4757	< 0.001	-0.3123	1.2557	
Subtropical moist forest	8829	0.7183	0.4572	< 0.001	-0.3128	1.2602	
Subtropical wet forest	3428	0.7263	0.4413	< 0.001	-0.2200	1.2392	
Lower montane	286	0.3646	0.5815	< 0.001	0.9809	0.7950	
Mangrove	88	0.4822	0.4557	< 0.001	0.4157	0.9727	
RMSE = root mean square error							

Table 4: Model statistics and parameter estimates from DBH prediction Equation 4a by Holdridge life zone and tree measurement protocol.

| | | | $\ln(DBH) = b_0 + b_1 \ln(R_{MAX})$ | | | | |
| | | | Model Statistics | | | Parameter Estimates | |
Life zone	Protocol	N	r^2	RMSE	Pr>F	b_0	b_1
All	All protocols	2791	0.2876	0.3699	< 0.001	2.4071	0.4720
	FIA	2600	0.1796	0.3392	< 0.001	2.5853	0.3410
	UFORE	191	0.7741	0.4023	< 0.001	1.7215	1.0655
Subtropical moist forest	All protocols	1609	0.3183	0.3879	< 0.001	2.3706	0.4945
	FIA	1418	0.1466	0.3337	< 0.001	2.6582	0.2846
	UFORE	191	0.7741	0.4023	< 0.001	1.7215	1.0655
Subtropical dry forest	FIA	225	0.1575	0.2780	< 0.001	2.5452	0.3045
Subtropical wet forest	FIA	746	0.1882	0.3427	< 0.001	2.5906	0.3677
Lower montane	FIA	118	0.4398	0.3071	< 0.001	2.2753	0.6447
Mangrove	FIA	93	0.3458	0.3983	< 0.001	2.0523	0.6723
RMSE = root mean square error							

Table 5: Model statistics and parameter estimates from DBH prediction Equation 5a by Holdridge life zone and tree measurement protocol.

| | | | $\ln(DBH) = b_0 + b_1 \ln(H_T) + b_2 \ln(R_{MAX})$ | | | | | |
| | | | Model Statistics | | | Parameter Estimates | | |
Life zone	Protocol	N	r^2	RMSE	Pr>F	b_0	b_1	b_2
All	All protocols	2739	0.3734	0.3229	< 0.001	1.8218	0.3063	0.3229
	FIA	2552	0.2305	0.2971	< 0.001	2.1422	0.2224	0.2391
	UFORE	187	0.8018	0.3639	< 0.001	1.2077	0.7678	0.3723
Subtropical moist forest	All protocols	1585	0.4068	0.3456	< 0.001	1.7754	0.3251	0.3283
	FIA	1398	0.1898	0.3025	< 0.001	2.2790	0.2000	0.1938
	UFORE	187	0.8018	0.3639	< 0.001	1.2077	0.7678	0.3723
Lower montane	FIA	115	0.4226	0.2798	< 0.001	1.9399	0.4700	0.2173
Subtropical dry forest	FIA	225	0.1803	0.2748	< 0.001	2.3566	0.2827	0.1032
Subtropical wet forest	FIA	735	0.3413	0.2849	< 0.001	1.5288	0.2100	0.4868
Mangrove	FIA	79	0.1939	0.2428	0.0003	2.5810	0.3073	-0.0053
RMSE = root mean square error								

Table 6: Model statistics and parameter estimates from DBH prediction Equation 5b by Holdridge life zone and tree measurement protocol.

| | | | $DBH = e^{b_0} * H_T^{b_1} * R_{MAX}^{b_2}$ | | | | | |
| | | | Model Statistics | | | Parameter Estimates | | |
Life zone	Protocol	N	r^2	RMSE	Pr>F	b_0	b_1	b_2
All	All protocols	2739	0.3409	7.1465	< 0.001	1.6284	0.4090	0.3272
	FIA	2552	0.2822	7.1187	< 0.001	1.6715	0.4028	0.3039
	UFORE	187	0.7419	6.3029	< 0.001	1.6044	0.2069	0.8123
Subtropical moist forest	All protocols	1585	0.3475	7.2541	< 0.001	1.7145	0.3674	0.3376
	FIA	1398	0.2445	7.2020	< 0.001	1.8163	0.3475	0.2948
	UFORE	187	0.7419	6.3029	< 0.001	1.6044	0.2069	0.8123
Lower montane	FIA	115	0.4826	7.1210	< 0.001	1.4340	0.3983	0.5585
Subtropical dry forest	FIA	225	0.2057	6.2519	< 0.001	2.1023	0.1714	0.3983
Subtropical wet forest	FIA	735	0.3707	7.0686	< 0.001	1.0804	0.6513	0.2543
Mangrove	FIA	79	0.1893	4.9000	< 0.001	2.5657	0	0.3359
RMSE = root mean square error								

icao) forest life zones with an overall R^2 value of 0.795. Perez (1970) also modeled crown diameter by DBH for trees in Puerto Rico and Dominica but did so based on the means of 10-cm diameter classes rather than on the DBH of the individually measured trees. By doing so, tree allometric variation was reduced resulting in uncommonly high R^2 values of 0.8510 to 0.9898 that are not analogous to the results of this or other studies cited herein. Bechtold [20] presented species-specific models predicting crown width based on DBH for 66 broadleaf species in temperate forests in the eastern U.S. R-square values ranged between 0.13 and 0.88 across all 66 species, with 36 species having R^2 values greater than or equal to 0.5.

There were substantial differences in our model fits by measurement protocol, with models fit to the UFORE data generally being much better than those fit to the FIA data. This could be for two reasons, the first biological and the second procedural. First, the urban forest trees in the UFORE data could possibly have more symmetrical, less variable crowns than their closed forest counterparts measured on the FIA plots. Basal areas on the UFORE plots ranged from 1.2 to 5.3 m^2/ha with an average of 3.1 m^2/ha (unpublished data), whereas basal areas on the FIA plots ranged from 8 to 26 m^2/ha with an average of 19 m^2/ha in Puerto Rico [24], and from 10 to 19 m^2/ha with an average of 13 m^2/ha for the U.S. Virgin Islands ..25]. Less competition on the UFORE plots allows the trees to grow fuller, more symmetrical crowns that are more amenable to modeling. Secondly, the FIA protocol called for the specific measure of the longest crown radius whereas the UFORE protocol measured along the cardinal directions with no regard for which part of the crown was widest or narrowest. Only by random chance would the longest axis of the crown be measured by the UFORE protocol and therefore, the variation in R_{MAX} for any given DBH was inherently smaller among the UFORE trees than among the FIA trees. On the FIA plots, variation in R_{LONG} may be exaggerated by the inclusion of atypically long, stray branches growing toward canopy gaps.

Modeling broadleaf tree crowns, particularly in the tropics, is complicated by the inherently high variability in crown width. In addition to the usual stand competition factors present in all forests, subtropical Caribbean forests experience hurricanes with sufficient frequency that trees potentially have their crowns damaged multiple times during their lifetime. This likely produces crowns that are reduced in size and more irregular for a given DBH than trees undamaged by hurricanes ....3]. Although we made every effort to exclude damaged stems and crowns from the data set, influential damages from the past are not always evident. The extent to which variability was compounded by past damage and the crown width measurement protocols is unknown. The crown width measurement protocols with which our data were collected was unlike that in other similar studies, i.e., that of using the average of two diameters, the first measured at the widest point of the crown and the second measured perpendicular to, and bisecting, the first [20, 21, 23, 26-28]. Perhaps variation would have been reduced, particularly for the FIA trees, had the data been collected in this more common manner.

It should also be remembered that models to estimate DBH from remotely-sensed crown and tree height measurements could potentially differ from models built using ground-based measurements and introduce additional sources of error. Asner et al. .8] describe the difficulties of estimating crown width from IKONOS imagery. Their satellite image-based crown area estimates were an average of 65% greater than field measurements.

5 Conclusions

Models that use a field or remotely-sensed measurement of H_T as a predictor variable can be expected to produce a reasonably accurate estimate of DBH in Caribbean subtropical moist and subtropical wet forest, but these estimates should only be used with an understanding of their limitations. Models that use a crown width measurement such as R_{MAX} are sensitive to field data collection methods and suffer from the variability inherent in tree crowns. With most R^2 values falling below a reasonably moderate level of correlation, the potential for error propagation from using a DBH predicted from R_{MAX} measurements in Caribbean forest biomass and carbon estimation models, as has been attempted for some Amazonian forests ..8, 9], is too great to earn our recommendation. While we would like to see the predictive capabilities of these models improved, we do not think that more data should necessarily be collected with the crown width measurement protocols employed here. We expect the use of other measurement protocols, such as measuring multiple radii from the bole to the drip line or measuring the longest diameter drip line to drip line and a perpendicular width, might provide data more amenable for modeling purposes. Therefore, we recommend further study of crown width measurement protocols to determine if indeed the irregular crowns of Caribbean forest species can be predicted with acceptable levels of accuracy. Also, we recommend that models that predict DBH from remotely-sensed crown and height measurements be developed for comparison to models derived from ground-based measurements.

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