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ALGEBRAIC DIFFERENCE EQUATIONS FOR STAND HEIGHT, DIAMETER, AND VOLUME DEPENDING ON STAND AGE AND SITE FACTORS FOR ESTONIAN STATE FORESTS

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Abstract. Algebraic difference equations of stand height, diameter, and volume depending on dominant species and site factors have been explored on the basis of Estonian state forest inventory data. Stand variables such as total age, average height, breast height diameter, volume, origin (naturally regenerated or cultivated), forest site type and dominant species from forest inventory database files of Estonian state forests have been used as initial data for this study. A total of 171 data series of height, diameter and volume on age were calculated as averages of data groups by site type, dominant species, origin, and age classes of 5 years. The Cieszewski and Bella (1989) algebraic difference equation has been used for model construction. First, tree parameters of the Hossfeld function were estimated for each of the height, diameter and volume series and relationships between the parameters were later studied. In the final model, dominant tree species, thickness of organic layer of soil, stand origin, height, diameter, and volume at given age were used as input variables. The model is included in the Estonian state forest information system and in several software packages for forest inventory data processing.

Keywords: growth modeling, algebraic difference equation, height, diameter, volume

1 Introduction

The republic of Estonia lies in Eastern Europe between latitudes 57^{o}30' and 59^{o}49' N and longitudes 21^{o}46' and 28^{o}13' E. The total area of forestland is 2.27 million ha, i.e. 51.9% of the area of Estonia. The volume of growing stock on forestland is 451 million cubic meters and is showing a trend to increase. Pine stands have the largest area and growing stock (710 thousand ha and 151 million m^3) while birch stands take second place (707 thousand ha and 118 million m^3) and spruce stands take third place (404 thousand ha and 87 million m^3) (Yearbook Forest 2004, 2005).

Phytogeographically, Estonia belongs to the northern part of the sub-belt of the nemoral coniferous or so-called mixed forests of the Northern Hemisphere's temperate zone forest belt (Etverk et al 1995). The soils are very diverse due to big differences in parent material and in relief, as well as in the length of soil genesis and to a lesser extent in climatic conditions.

As a consequence of all this, forests of Estonia vary on a very large scale: there are dark boreal spruce forests with treetops at a height of 40 meters; heath pine forests, stunted in growth but full of sunshine; unique alvar forests growing on a layer of soil that is only a few centimeters thick and lies on a stratum of limestone rock; and wet bog forests on peat layers several meters thick.

An ordinated forest typological classification (Lõhmus 1984) has been worked out. The set of Estonian forest site types is presented in Table 1. According to the dominant tree species there can be either one or several forest types in each site type.

Until recent times the forest growth and yield tables of Estonia and its nearest neighbors have been used in Estonia to offer predictions of forest growth. However, Estonian forests are quite variable and the several growth tables of differing quality could not describe this variability well enough. Thus there has been a growing need for more general forest growth models.

From the aspect of modeling of Estonian forest growth, stand descriptions of state forest inventory are the most reliable data available in the state forest databases. Traditionally, state forest inventories take place every 10 years. During the forest inventory, ocular estimates of most stand variables (species composition, site type, stand age, height, diameter, volume, etc.) are assessed for each sub-compartment.

The purpose of the present study is to explore a model

Table 1: Estonian forest site types by E. Lõhmus (1984) and thickness of organic layer of soil (OHOR).

Code	OHOR cm	Site type	Code	OHOR cm	Site type
LL	2	*Arctostaphylos-alvar*	SL	1	*Hepatica*
KL	1	*Calamagrostis-alvar*	ND	1	*Aegopodium*
SM	4	*Cladonia*	SJ	15	*Dryopteris*
KN	5	*Calluna*	AN	10	*Filipendula*
SN	20	*Vaccinium uliginosum*	TAN	15	*Carex-Filipendula*
PH	4	*Rhodococcum*	OS	20	*Equisetum*
JPH	4	*Oxalis-Rhodococcum*	TR	20	*Carex*
MS	10	*Myrtillus*	RB	50	*Raised (oligotrophic) bog*
JMS	6	*Oxalis-Myrtillus*	SS	50	*Transitional (mesotrophic) bog*
KMS	13	*Polytrichum-Myrtillus*	MDS	50	*Alder-birch (eutrophic-mesotropic) swamp*
KR	20	*Polytrichum*	LD	50	*Alder (eutrophic) fen*
JK	4	*Oxalis*	KS	50	*Drained swamp*

for prediction of the growth of stand height, diameter and volume using the present state of the stand and site variables.

2 Materials

As initial data for modeling, stand records of Estonian state forest inventory in 1984-1993 were used (Kiviste 1995, Kiviste 1997). Average height, mean squared breast height diameter, and volume of 423,919 stands were grouped by dominant tree species (Table 2), forest site type (Table 1), stand origin (naturally regenerated or cultivated), and stand age (using 5-year intervals). Data from very young stands (age below 20 years for coniferous and hardwood, and 10 years for deciduous forests), from over-matured stands and outliers were excluded before the calculation.

The minimum and maximum age for stand selection, the number of series and the number of stands by dominant species and stand origin are presented in Table 2.

As the result of grouping, a total of 171 age-series of height, diameter, and volume were obtained. For illustration, empirical height series of pine, spruce, birch, and aspen stands are presented in Figures 1-4.

For evaluation of the model, data from the network of forest growth permanent sample plots in Estonia were used. The network of permanent sample plots was established in 1995–2004. By 2005 the first re-measurement data had been obtained from 380 sample plots. Of those, 93 sample plots were thinned during the period between measurements. The design and method of establishing and measuring permanent sample plots is described by A. Kiviste and M. Hordo (2002).

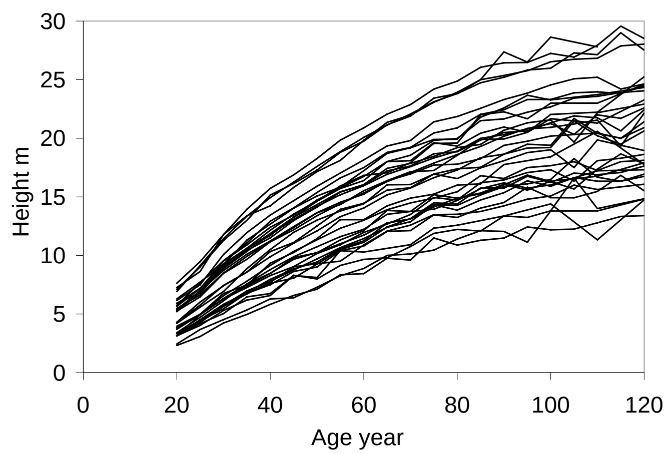

Figure 1: Height series of pine stands by forest site type.

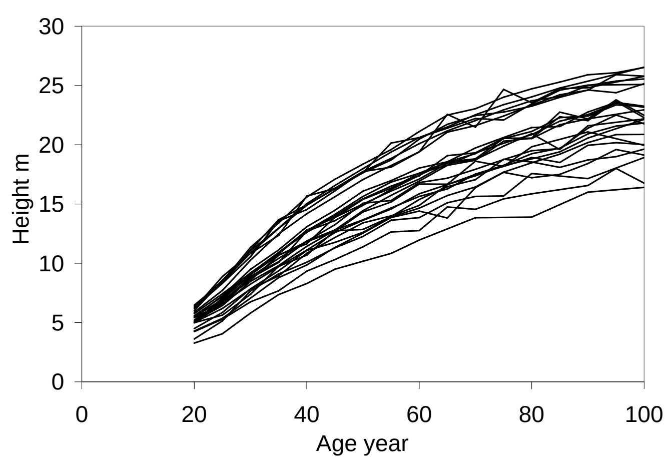

Figure 2: Height series of spruce stands by forest site type.

Table 2: Maximum and minimum ages for stand selection, number of age-series and number of stands used for construction of series by dominant species and by stand origin (K – code of origin used in the model).

Species (Sp)	Origin	K	Min. age	Max. age	No. series	No. stands
Scots pine	Naturally regenerated	0	20	120	34	151710
Scots pine	Planted	1	20	120	28	54659
Norway spruce	Naturally regenerated	0	20	100	25	55730
Norway spruce	Planted	1	20	100	11	21180
Silver birch and downy birch	Naturally regenerated	0	10	70	30	118080
Aspen	Naturally regenerated	0	10	60	9	6557
Common alder	Naturally regenerated	0	10	50	15	5170
Grey alder	Naturally regenerated	0	10	50	9	9254
Common oak	Naturally regenerated	0	20	120	3	822
Common oak	Planted	1	20	120	3	205
Common ash	Naturally regenerated	0	20	100	4	552

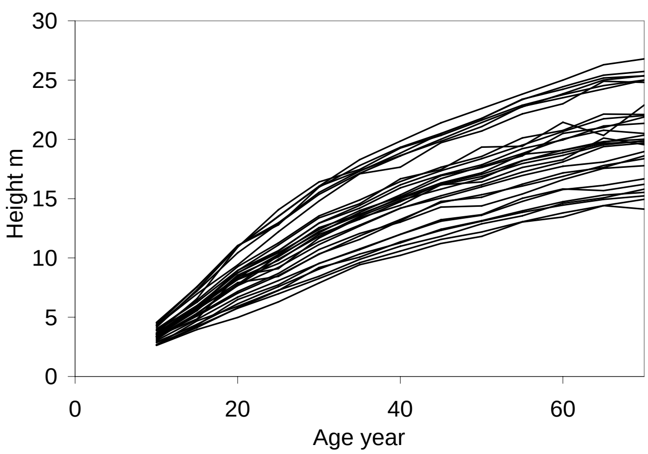

Figure 3: Height series of birch stands by forest site type.

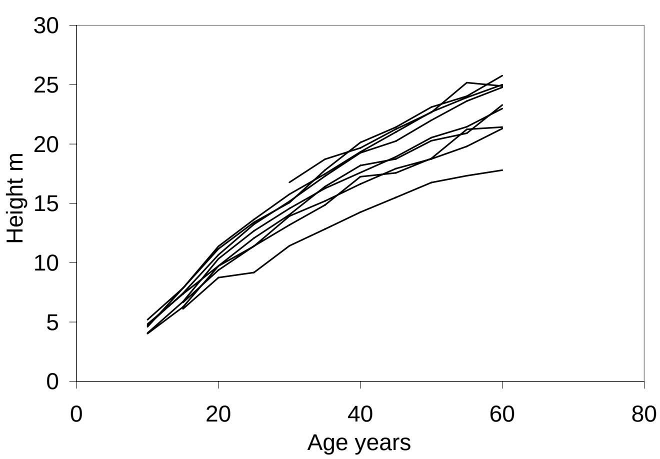

Figure 4: Height series of aspen stands by forest site type.

3 Methods

In forest growth modeling, equations for predicting stand variables, for example height, are expressed usually in the general form

$$H = f(A, H_B,) \tag{1}$$

where: H is height of the stand at age A; and H_B is height of the stand at a base age B (site index).

In case we know stand height H_1 at age A_1, then for height prediction using equation (1), site index H_B should be calculated first. Site index H_B can be obtained by solving the following equation:

$$H_1 = f(A_1, H_B). \tag{2}$$

In most cases for solving equation (2), iteration methods are necessary. To reach a solution of necessary precision a few iteration steps are usually enough. However, iteration programming is quite a time-consuming and complicated task. In certain cases the iteration method may not converge. This disadvantage does not occur in the case of algebraic difference equations given in the general form

$$H_2 = g(A_1, H_1, A_2), \tag{3}$$

where: H_2 is the predicted stand height at any age A_2; and H_1 is the known stand height at given age A_1.

The majority of the stand growth algebraic difference equations have been derived on the basis of rather simple growth functions (Clutter et al. 1983, Rayner 1991, Rennolls 1993), which are not flexible enough for predicting the growth of height, diameter, and volume (Kiviste 1988, Kiviste et al 2002). Derivation of algebraic difference equations from superior growth functions is usually algebraically impossible.

A smart and interesting solution is the algebraic difference equation by Cieszewski and Bella (1989) for the Hossfeld growth function. Hossfeld's growth function is known in the form of

$$H = \frac{b_0}{1 + \frac{b_1}{A^{b_2}}}, \quad (4)$$

where: H is stand height at the age A, and b_0, b_1, and b_2 are the growth function parameters.

Supposing that the growth function passes a point of base age (B , H_B), the function (4) can be presented as

$$H = \frac{H_B \cdot \left(1 + \frac{b_1}{B^{b_2}}\right)}{1 + \frac{b_1}{A^{b_2}}}, \quad (5)$$

where: H_B is stand height at a base age B (site index), and b_1 and b_2 are growth function parameters.

Cieszewski and Bella (1989) learned that parameters H_B and b_1 of the growth function (5) are inversely proportional.

$$b_1 = \frac{\beta}{H_B}. \quad (6)$$

Replacing the parameter b_1 in the equation (5) with the relation (6) we get

$$H = \frac{H_B + \frac{\beta}{B^{b_2}}}{1 + \frac{\beta/H_B}{A^{b_2}}}. \quad (7)$$

Substituting the base age B and site index H_B in equation (7) with the variables A_1 and H_1 the following algebraic difference equation is obtained (Cieszewski & Bella 1989).

$$H_2 = \frac{H_1 + d + r}{2 + \frac{4 \cdot \beta}{(H_1 - d + r) \cdot A_2^{b_2}}}, \quad (8)$$

where: $d = \frac{\beta}{B^{b_2}}$, and

$$r = \sqrt{(H_1 - d)^2 + \frac{4 \cdot \beta \cdot H_1}{A_1^{b_2}}}.$$

The difference equation (8) proved to be appropriate for modeling of dominant height growth of pine forests in Sweden on the basis of permanent sample plot data (Elfving & Kiviste 1997), for modeling of dominant height growth of birch forests in Sweden on the basis of tree increment core data (Eriksson et al 1997) and in other studies (Trincado et al 2003, Kasesalu & Kiviste 2001). These successful experiences encouraged us to use the same difference equation (8) for modeling the Estonian height, diameter, and volume series.

3.1 Estimating the model parameters. The algebraic difference equation (8) includes three arguments A_1, H_1 and A_2 and three parameters B, β and b_2. To estimate the parameters of a difference equation, the stand growth data are usually presented as set of intervals $\{(A_1, H_1), (A_2, H_2)\}$. In previous studies the parameter B (base age) was fixed by trial and error (Elfving & Kiviste 1997, Eriksson et al 1997, Trincado et al 2003). For Estonian data we fixed the value 50 years for base age B. Parameters β and b_2 were estimated using the procedure of non–linear regression analysis on the interval data.

In site index models (Cieszewski & Bella 1989, Elfving & Kiviste 1997, Eriksson et al 1997) parameters β and b_2 were considered as constants for each tree species for a certain geographical region. In that case model (8) presents a one-parameter set of growth curves upon the age/height plane. According to our previous studies (Kiviste, 1995) the growth curves depend on site index and on the site properties (thickness of organic layer). Thus the height, diameter and volume should be modelled as a two-parameter set of curves.

In this study we used a combined method. In the first modeling step of, a total of 171 age-series of height, diameter, and volume were approximated using the three-parameter Hossfeld function (4). Using the non-linear regression procedure NLIN of SAS software (SAS Institute Inc. 1989) a set of parameters b_0, b_1, and $b_2$were estimated for each height (H), diameter (D), and volume (M) series. To distinguish the set of parameters we added a letter respectively, for example bH_2 is the parameter b_2 for mean height. For this model residual standard errors of 0.48 m, 0.66 cm, and 12.4 m^3ha^{-1} were estimated in relation to the height, diameter and volume series, respectively.

The Analysis of Variance proved significant difference of the parameter bH_2 by tree species. The variability of the parameter bH_2 by tree species is represented in the Figure 5. The different values of parameter b_2 by tree species were observed on the box-plots of diameter and volume as well. Medians of parameter b_2 for height, diameter and volume were estimated for each species (Table 3).

In the second modeling step, the inversely proportional relation of parameters bH_1, bD_1 and bM_1 and variables H_{50}, D_{50} and M_{50} were studied. The scatter-plot of the parameter bH_1 against the site index H_{50} (Figure 6) shows large variation of the parameter bH_1. Nevertheless, the inversely proportional relationship by species could be observed in the scatter-plots for height (Figure 6), diameter, and volume.

Next, coefficients βH, βD and βM for each age-series were calculated using equation (6). We studied the relationship of the coefficients βH, βD and βM on forest type variables: dominant tree species, thickness of organic layer of the soil, drainage and the origin of the stand. Among these variables the thickness of organic

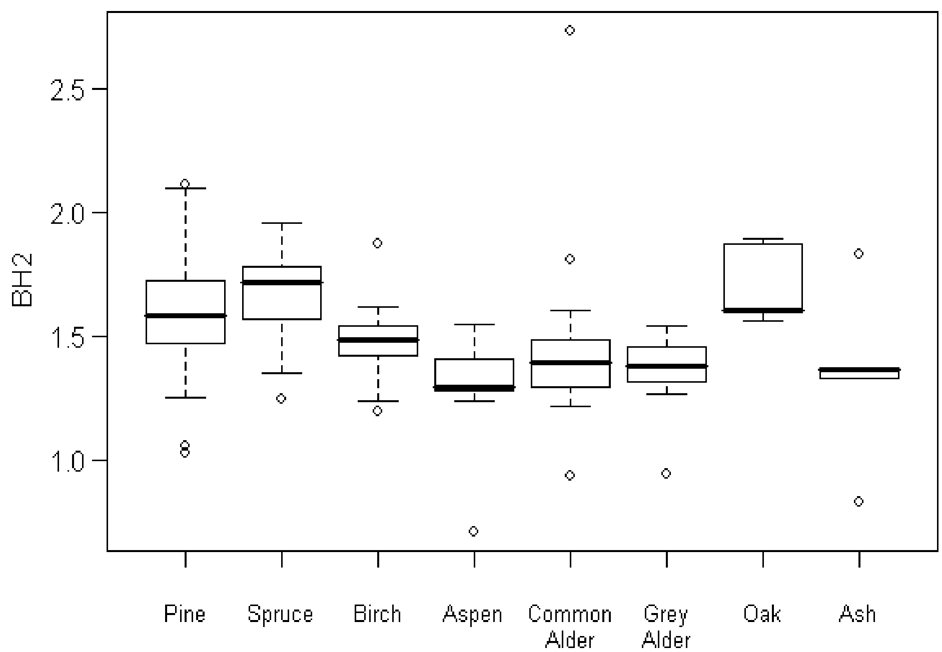

Figure 5: Box-plots of parameter bH_2 by dominant species. bH_2 = parameter b_2 for mean height.

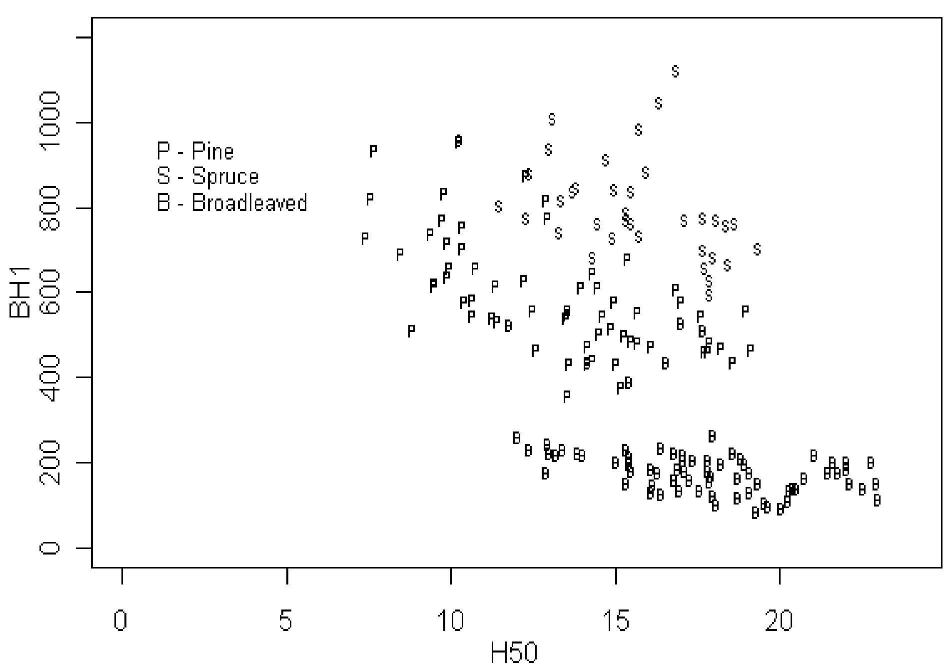

Figure 6: Scatter-plot of parameters bH_1 and site index H_{50} by species. bH_1 = parameter b_1 for mean height.

layer is continuous, the other are discrete variables. For the covariance analysis, the procedure of general linear methods (GLM) of the SAS software (SAS Institute Inc. 1989) was used. In the analysis, every series was weighted proportionally with the number of stands and inversely proportional with the residual variance calculated at the first step of modeling. Only the significant variables (tree species, thickness of organic layer and origin of the stand, significance level $\alpha = 0.05$) were included in the model. The origin of the stand was set to 0 when the stand was cultivated (seeded or planted) and 1 when the stand was naturally regenerated. The following linear model was obtained:

$$\beta = C_0 + C_1 \cdot ln(OHOR + 1) + C_2 \cdot K, \qquad (9)$$

where: OHOR is the thickness of organic layer of soil cm; K is the dummy variable (Table 2); C_0 is a constant depending on tree species; and C_1, C_2 are other model constants.

4 Results

An algebraic difference equation was explored for predicting stand height (H_2), breast height diameter (D_2) and volume (M_2) at any age (A_2) on the basis of the present state of stand description data (A_1, H_1, D_1, M_1). The algorithm is the following.

1. Determine the input values of the model:

- dominant tree species (pine, birch, spruce, aspen, grey alder, common alder, oak or ash);
- thickness of the organic layer of soil OHOR cm (Table 1);
- origin of the stand K (Table 2);
- stand age at a given moment A_1 years;
- stand height (H_1) m, diameter (D_1) cm or volume (M_1) m^3ha^{-1} at a given moment;

2. Find the constants bH_2, bD_2, bM_2, CH_0, CD_0, and CM_0 according to the dominant tree species (Table 3).

3. Calculate the coefficients ßH, ßD, and ßM of the equation (9) (LN is a function of the normal logarithm).

ßH = CH_0 - 493·LN(OHOR + 1) + 1355·K; (10)

ßD = CD_0 - 306·LN(OHOR + 1); (11)

ßM = CM_0 - 54348·LN(OHOR + 1) + 56290·K. (12)

4. Calculate the variables dH, rH, dD, rD, dM and rM (SQRT is a function of the square root):

dH = ßH/50^{bH2} (13)

rH = SQRT((H_1 - dH)^2 + 4·ßH·H_1/A_1^{bH2}); (14)

dD = ßD/50^{bD2} (15)

rD = SQRT((D_1 - dD)^2 + 4·ßD·D_1/A_1^{bD2}); (16)

dM = ßM/50^{bM2} (17)

rM = SQRT((M_1 - dM)^2 + 4·ßM·M_1/A_1^{bM2}). (18)

5. Calculate the predicted height (H_2, m), diameter (D_2, cm), and volume (M_2, m^3ha^{-1}) at desired age A_2.

H_2 = (H_1 + dH+ rH)/(2 + 4·ßH·A_2^{-bH2}/(H_1 - dH + rH)), (19)

D_2 = (D_1 + dD + rD)/(2 + 4·ßD·A_2^{-bD2}/(D_1 - dD + rD)), (20)

M_2 = (M_1 + dM + rM)/(2 + 4·ßM·A_2^{-bM2}/(M_1 - dM + rM)). (21)

6. The algebraic difference model (10)–(21) can also be used for site index calculation. In this case the base age of site index (for example 100 years) should be assigned to argument A_2.

7. If we know the values of parameters H_{50}, D_{50}, and M_{50} for a certain site type then stand height, diameter, and volume can be predicted using the following equations. Average values of parameters H_{50}, D_{50}, and M_{50} for most Estonian forest types are presented in the worksheet "Andmed" of MS Excel file (http://www.eau.ee/∼akiviste/kktab2.xls).

H = (H_{50} + βH/50^{bH2})/(1+(βH/H_{50})·A^{-bH2}), (22)

D = (D_{50} + βD/50^{bD2})/(1+(βD/D_{50})·A^{-bD2}), (23)

M = (M_{50}+βM/50^{bM2})/(1+(βM/M_{50})·A^{-bM2}). (24)

The difference model (10)–(21) was fitted to the height, diameter, and volume series by finding the most suitable values of H_{50}, D_{50} and M_{50} for each series. Predictions for series were calculated from the state A_1 = 50, H_1 = H_{50}, D_1 = D_{50} and M_1= M_{50}. No significant bias between predictions and height, diameter, and volume series were found. The residual standard errors 0.57 m, 0.83 cm, and 17.0 m^3ha^{-1} of the model were calculated in relation to the height, diameter and volume series. These residual standard errors were slightly higher than those in the case of the Hossfeld function (4).

5 Model evaluation

The algebraic difference model (10)–(21) was evaluated on 287 permanent-sample plot data measured twice with an interval of 5 years in 1995–2004. The plots were located randomly in different parts of Estonia and the stands were not thinned between the two measurements. Also, plots with great mortality caused by natural disturbances were excluded from the analysis.

Data from the first measurement of plots (stand age, height, diameter, volume, thickness of organic layer of soil, stand origin, dominant species) were assigned to input variables of the model. Using the difference model, stand height, diameter, and volume were predicted five years forward and compared with the plot re-measurement data.

In Figures 7, 8, and 9 the differences between the re-

Table 3: Parameter estimates for height, diameter and volume algebraic difference equations.

Species	bH_2	bD_2	bM_2	CH_0	CD_0	CM_0
Pine	1.58	1.33	1.93	8319	6051	380544
Spruce	1.71	1.54	2.20	12867	9805	875924
Birch	1.48	1.37	2.05	4990	5034	446641
Aspen	1.30	1.15	1.77	3882	7092	310877
Common alder	1.41	1.41	1.93	4228	4438	378317
Grey alder	1.38	1.35	1.78	2749	2864	205882
Oak	1.61	1.45	2.02	6742	10509	277948
Ash	1.35	1.03	2.12	3732	5405	345440

sults of the second measurements and predicted values for stand height (EH), diameter (ED), and volume (EM) are presented.

No overall bias of height and diameter growth predictions can be observed in Figures 7 and 8. The residual standard errors of five-year height and diameter growth predictions were 0.86 m and 0.58 cm respectively. However, at young ages the model slightly overestimates and at mature ages underestimates the actual growth of height and diameter.

Such trends were not revealed when difference model predictions were compared with initial data (height, diameter, and volume series compiled from forest inventory data). However, a similar effect became evident when site indices of forest inventories in the 1950s and 1990s were compared (Kiviste, 1999). This trend could be explained by the hypothesis that forest growth conditions were improved and the stand growth was accelerating in Estonia during the last decades (Nilson & Kiviste 1986).

Figure 9 shows that volume growth predictions are on an average 20 m^3ha^{-1} lower than their actual values by the permanent plot data. Apparently, this could be caused by the fact that thinned and seriously damaged stands were excluded from the comparison while most Estonian forest stand data (including thinned and damaged stands) was used for model building. The residual standard error of five-year volume growth predictions was 15 m^3ha^{-1}.

6 Discussion

In this study, a system of algebraic difference equations for prediction of stand height, diameter, and volume of Estonian forests have been explored. The model summarizes large amounts of forest inventory data which is its major advantage in comparison with previous models and growth and yield tables used in Estonia. The model parameters cover a huge variety of forest site properties, which enables us to generalize forest growth for different forest site types using a smart system of equations.

The structure of the model expressed in the form of algebraic difference equations is a convenient way of using it and enables its easy employment in applications. The algebraic difference model (10)–(21) proved to be reliable and trouble-free and that is one reason why it is included into the Estonian state forest information system and into several software packages for forest inventory data processing.

The model (10)–(21) describes most reliably the growth of dominant forest types from the age of pole forests up to the age of matured forests. As a rule, model extrapolation beyond the range of initial data is not recommended. However, the basic function of the model is a classical Hossfeld growth function, which is one of the most suitable functions for forest growth modeling (Kiviste 1988, Kiviste et al 2002); thus we should obtain reasonable extrapolations even for young stands and over-matured stands.

Upon approximation of the height, diameter and volume series, the number of stands was used as the weight of each observation. Therefore, the regularities of the most widely spread species like pine, spruce, and birch, and dominating site types like *Myrtillus, Rhodococcum*, and *Aegopodium* have been "imposed" on relatively uncommon species and site types. Using the weight function for modeling relationships between parameters was necessary because series of rare forest types having a relatively low number of observations appeared too "erratic" to detect any regularities.

The set of input parameters of difference model (10)–(21) does not contain several influential variables like stand density, stand composition, forest management schedules, etc. That is why volume growth in unthinned permanent plots was on an average higher than predicted. However, adding new variables into the model seems to be quite sophisticated while forest inventory data only are used for model building. For building a more detailed stand growth model a huge amount of data

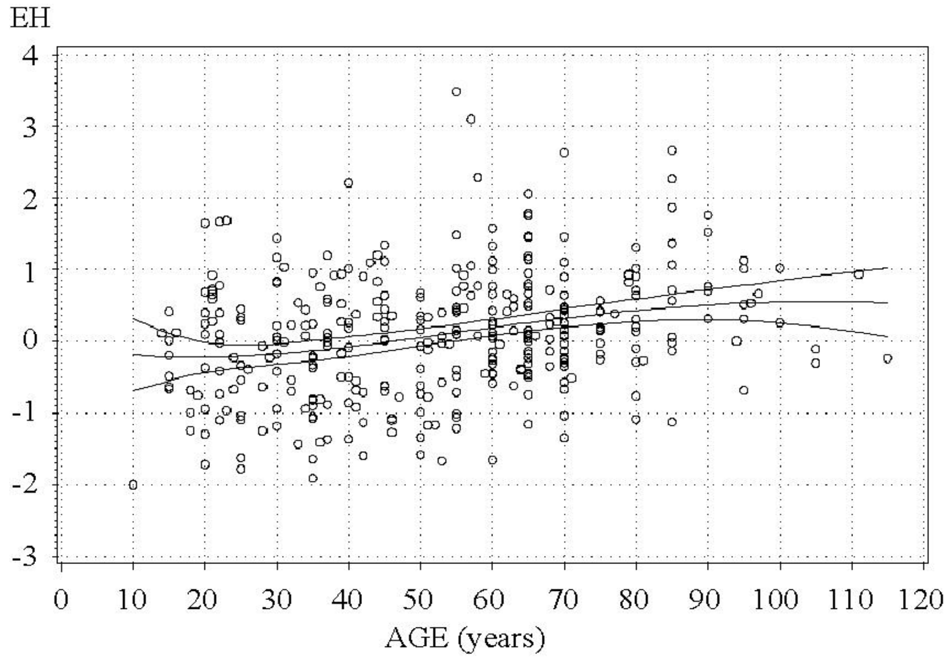

Figure 7: Errors in meters of the difference model for predicting five-year stand height growth, depending on stand age at first measurement. The trend curves show model bias and its 95% confidence limits. EH = H_2actual – H_2model

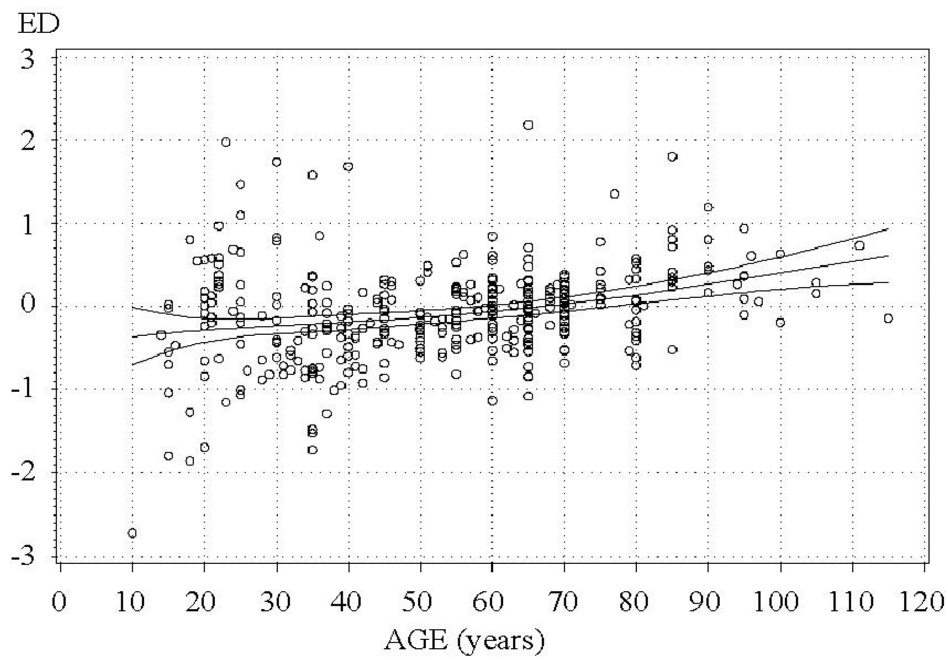

Figure 8: Errors in centimeters of the difference model for predicting five-year diameter growth, depending on stand age at first measurement. The trend curves show model bias and its 95% confidence limits. ED = D_2actual – D_2model.

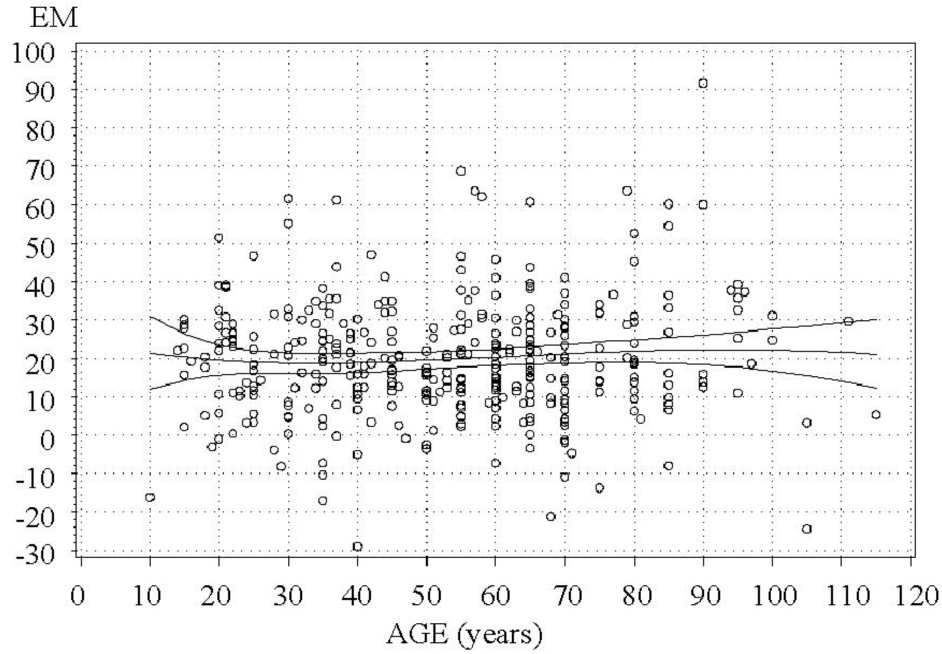

Figure 9: Errors in m^3ha^{-1} of the difference model for predicting five-year volume growth, depending on stand age at first measurement. The trend curves show model bias and its 95% confidence limits. EM = M_2actual – M_2model.

from a well-designed set of permanent plots should be used.

The model is based on Estonian state forest inventory data collected in 1984–1993, grouped by forest type and age class. Those series express the relationship of how conditional average values of height, diameter, and volume are depending on stand age. These series can coincide with the real growth of the stands (assuming that the ocular estimates of forest surveyors are free of systematic errors) only when the growth conditions of the stands have been stable in time. Several studies, however, point at changing growth conditions, demonstrating a considerable increase in forest growth during recent decades (Nilson & Kiviste 1984, Eriksson & Johansson 1993, Elfving & Tegnhammar 1996). In that case the model offered in this paper will actually give a bit smaller predict than realistic prognoses.

7 Conclusion

A method for construction of an algebraic difference model from forest inventory stand description data has been presented in this paper. Using this method, system of algebraic difference equations (10)–(21) have been explored for predicting stand height, diameter, and volume growth on the basis of the present state of stand description data.

As initial data for this study, stand variables like total age, average height, diameter, volume, stand origin, site type and stand composition by species from database files of all Estonian state forest districts have been used. Height, diameter and volume series on age were calculated as averages of data groups by site type, by dominant species, by origin and by age classes of 5 years. A total of 171 data series has been created from 423,919 stand descriptions.

The Cieszewski, Bella (1989) algebraic difference equation (8) has been used for model construction. First, tree parameters of Hossfeld function (4) were estimated for each of the height, diameter and volume series, and later relationships between the parameters were studied.

Finally, an algebraic difference equation model (10)–(21) has been developed. Dominant tree species (Table 2), thickness of organic layer of soil (Table 1), stand origin (Table 2), height, diameter, and volume at given age were used as input variables of the model. Parameter estimates of the model are presented in Table 3 and in equations (10)–(12).

No significant bias between model predictions and initial data (height, diameter, and volume series) were found. The residual standard errors 0.57 m, 0.83 cm, and 17.0 m^3ha^{-1} of the model were calculated in relation to the height, diameter and volume series.

The model (10)–(21) was evaluated on data from 287 permanent sample plots measured twice with an interval of five years in 1995–2004. The residual standard errors of five-year height and diameter increment predictions were 0.86 m and 0.58 cm respectively. However, at young ages the model slightly overestimates and at mature ages underestimates the actual growth of height and diameter. Volume growth predictions were on an average 20 m^3ha^{-1} lower than their actual values on the basis of the permanent plot data. This could be caused by the fact that thinned and seriously damaged stands were excluded from the comparison while most Estonian forest stand data (including thinned and damaged stands) was used for model building. The residual standard error of five-year volume increment predictions was 15 m^3ha^{-1}.

The structure of the model expressed in the form of algebraic difference equations is a convenient way of using it and enables its easy employment in applications. The model (10)–(21) proved to be reliable and trouble-free, which is one reason why it is included in the Estonian state forest information system and in several software packages for forest inventory data processing.

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