

# A TEST OF THE MEAN DISTANCE METHOD FOR FOREST REGENERATION ASSESSMENT

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**ABSTRACT.** A new distance-based estimator for forest regeneration assessment, the mean distance method, was developed by combining ideas and techniques from the wandering quarter method, T-square sampling and the random pairs method. The performance of the mean distance method was compared to conventional 4.05 square meter plot sampling through simulation analysis on 405 square meter blocks of a field surveyed clumped distribution and a computer generated random distribution at different levels of density of 100, 50 and 25%. The mean distance method accurately estimated density on the random populations but the mean distance method estimates were more variable than those of 4.05 square meter plot sampling. The mean distance method overestimated actual density and was less precise than plot sampling when both methods were tested on the clumped populations. The optimum sample sizes needed for the mean distance method to achieve the same precision as 4.05 square meter plot sampling at all three density levels, for both the random and clumped spatial distributions, were at least 10 times larger than the sample size used for 4.05 square meter plot sampling.

**Keywords:** Key Words: sampling, plot, distance, accuracy, seedlings.

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## 1 INTRODUCTION

Estimating the density of regeneration, or number of seedlings per unit area, on a given site is important to foresters for assessing existing regeneration, determining reforestation needs, and determining if reforestation efforts have been successful. A variety of sampling methods have been developed for estimating regeneration density. The majority of the methods fall into two general categories: plot sampling and distance sampling (Payandeh and Ek, 1986). Plot sampling is the traditional approach that involves establishing fixed size plots within an area, counting the trees within each plot, and then converting the tree counts to a density estimate. On the other hand, distance sampling involves measuring the distance(s) from a sample point or tree to another tree(s) within an area and then using this distance(s) to estimate density. Our objective was to compare the performance of a new distance-based method, known as the mean distance method, as an alternative to traditional plot sampling. Both methods were evaluated through computer simulation analysis on 405 square meter blocks (0.1 acre) of a field surveyed clumped distribution and a computer generated random distribution at different density levels of 100, 50 and 25%.

## 2 DISTANCE SAMPLING

Distance sampling has attracted the attention of researchers over the past 50 years as a means of estimating density. Its main attraction is that it is fast, easy to use, and one or more distances are always recorded at each sample point. In contrast, plot sampling can sometimes be a very time consuming process, boundary trees may be overlooked, and some plots may have no tallies.

Past attempts to develop a robust distance-based density estimator have not been very successful. In general, distance estimators are not robust and tend to be biased when the spatial distribution of the population under consideration does not represent a spatially random distribution (Persson, 1971). The lack of robustness is of concern because plants in natural populations tend to be aggregated, not distributed randomly (Patil et al., 1979). The major weakness of density estimators is that their bias is dependent on the spatial distribution of the population (Delince, 1986). Some distance estimators have been shown to be unbiased over a wide range of spatial patterns if the estimators are adjusted according to the spatial pattern, but this adjustment would necessitate additional tests to determine a population's spatial distribution before estimating density.

Pollard (1971) used a maximum likelihood method to estimate forest density from a random point using distances between points and provides examples of using two nearest trees to a random point where the standard deviation to the second tree is reduced by 30. He found that the number of sample points required to estimate density with a prescribed accuracy does not depend on the density being measured and estimates of large diameter tree densities were reliable. Hanberry et al. (2011) compared the Pollard (1971) and the Morisita (1957) methods for estimation of tree density by analyzing spacing methods and concluded that the Morisita estimator outperformed the Pollard estimator in non-random and clustered distribution under typical forest conditions. Although researchers have studied distance sampling in quantifying vegetation, no study to date has provided a distance based method of density estimation as an alternative to traditional plot sampling. This research was undertaken to reinvigorate the desire and need for a fast and efficient method of density estimation as an alternative to plot sampling.

This study evaluated a new distance-based density estimator that combines the attractive features of the wandering quarter method (Catana, 1963), T-square sampling (Aherne and Diggle, 1978; Diggle, 1975; Diggle, 1977) and the random pairs method (Cottam and Curtis, 1949; Cottam and Curtis, 1955). The performance of the new distance estimator, which was evaluated through simulation analysis, was proposed as a statistically robust, fast and easier estimator to implement than traditional plot sampling.

### 3 METHODS

The new method of density estimation, to be known as the mean distance method, incorporates the feature of making multiple distance measurements between trees in one general direction from the wandering quarter method (Catana 1963). Because most natural stands tend to be aggregated, directional measurement gives the estimator mobility and forces it out of clumped areas into open areas, or out of open areas into clumped areas, depending on the location of the sample point. By sampling through a population and not remaining stationary, the estimator is better able to determine the overall average distance between trees. This overall average distance is used to determine the average area occupied per tree. The inverse of the average area per tree is the density estimate which was incorporated from the random pairs method for ease of calculation. The feature of measuring the distance from one tree to its nearest neighbor across a 180 degree line was incorporated from T-square sampling to simplify locating seedlings in the field.

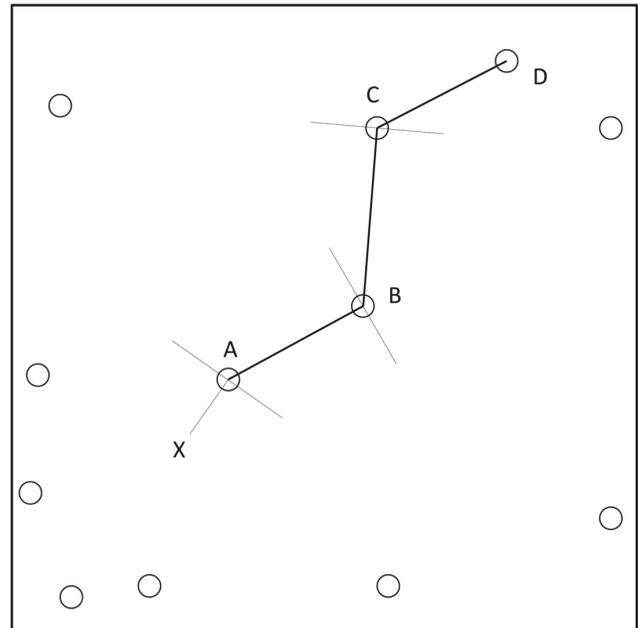


Figure 1: Procedure of the mean distance method assuming  $s$  equals three. The circles represent trees,  $X$  is a sample point, and  $A$  is the closest tree to sample point  $X$ . Measure line  $A - B$  because  $B$  is the nearest tree to  $A$  lying beyond a line drawn through  $A$  which is perpendicular to line  $X - A$ ; measure line  $B - C$  because  $C$  is the nearest tree to  $B$  lying beyond a line drawn through  $B$  which is perpendicular to line  $A - B$ ; and measure line  $C - D$  because  $D$  is the nearest tree to  $C$  lying beyond a line drawn through  $C$  which is perpendicular to line  $B - C$ .

The procedure (Fig. 1) for obtaining the necessary distances for the mean distance method at each sample point within a population follows:

1. Beginning with a randomly located sample point ( $X$ ), locate the closest tree ( $A$ ).
2. Measure the distance from the closest tree ( $A$ ) to its nearest neighbor ( $B$ ) lying beyond a line drawn perpendicular to the sample point-closest tree line (line  $X - A$ ) which intersects the closest tree ( $A$ ).
3. Next, measure the distance from the last tree measured ( $B$ ) to its nearest neighbor ( $C$ ) lying beyond a line drawn perpendicular to the last measured distance line (line  $A - B$ ) which intersects the last tree measured to ( $B$ ).
4. Continue measuring distances as described in step 3, up to  $s$  number of times.

Table 1: Seedlings per hectare of the computer generated random and field surveyed clumped spatial distributions at 100, 50 and 25% of density level.

Spatial Distribution	Distribution	Density Level (%)	Actual Density (seedlings/hectare)
Random		100	70642
Random		50	35321
Random		25	17661
Clumped		100	70617
Clumped		50	35296
Clumped		25	17636

By assuming the area occupied by a tree is represented by a hexagon and that the average of the distances measured represents the average distance between trees, one can apply the following formula discussed by Cottam and Curtis (1949, 1955) to estimate density at each sample point:

$$di = \frac{43560}{0.8661(si)^2}$$

where  $di$  = population density estimate per acre at sample point  $i$ ,  $si$  = average of the distances measured at sample point  $i$  in feet.

Density can then be estimated for the entire population by using the following formula:

$$D = \frac{\sum_1^n di}{n}$$

where  $D$  = estimated population density,  $n$  = number of sample points.

The estimated variance of the population ( $V$ ) is calculated by the following formula:

$$V = \frac{\sum_1^n (di - D)^2}{(n - 1)}$$

#### 4 DATA ANALYSIS AND RESULTS

The mean distance method was evaluated using computer simulation. Distances were measured to the nearest 0.254 centimeters (0.1 inch). Simulation allowed testing the mean distance method at different population densities and spatial distributions, and the determination of a reasonable value of  $s$ , the number of distances measured per sample point. The mean distance method was compared to 4.05 square meter plot sampling.

The first step of the simulation analysis involved obtaining clumped and random seedling populations on which to conduct the density estimation. Mapped

seedling data representing a clumped distribution of 2,859 seedling locations in a 405 square meter block within a 15-month-old clearcut in central Pennsylvania were used. An artificial random seedling distribution was generated in computer with 2,860 seedling locations within a 405 square meter block, representing the same density as the field surveyed data. These two distributions were further resampled to 50 and 25% population levels to represent 3 different populations representing 100, 50, and 25% of actual density to test the density estimator across variable population densities (Tab. 1). A Fortran based computer program called REGEN was written to simulate the mean distance method and the 4.05 square meter plot sampling. The program was written to allow the following parameters to be varied: (1) the density estimator, (2) the spatial pattern of regeneration, (3) the seedling density, (4) the number of replications, (5) the sample size, and (6) the number of distances measured per sample point for the mean distance method. The program calculates the average density estimate, average variance of the mean and average coefficient of variation over all replications of a given sample size.

REGEN was used in initial trials of the mean distance method to determine the optimum value of  $s$ , the number of distances measured per sample point. Twenty-five replications of sample size 30, with  $s$  ranging from one to ten, were tested on three densities in the random distribution (Tab. 2). The results from these initial trials were evaluated to determine the proportions that the mean distance method overestimated actual density of all three density levels of random distribution at all distances tested (Tab. 3). The mean distance method's overestimation of actual population density decreased as the number of distances measured per sample point increased, but remained approximately equal for a given density regardless of population density when the value of  $s$  was greater than one. The average variance of the mean for the mean distance method decreased as the number of distances measured per sample point increased, but was higher at any distance and density than the average variance of the mean for 4.05 square meter plot sampling when tested on the three densities in the random distribution with 500 replications of sample size 30 (Tab. 4). Average coefficient of variation (CV) for the 4.05 square meter plot ranged from 3 – 7%, with higher average CV's at lower densities (Tab. 4). By contrast, average CV's were 10 – 11% for the mean distance method at all densities if 5 distances were measured, and as low as 6 – 7% when up to 10 distances were measured (Tab. 2).

The overestimation factors were analyzed to choose the optimum value of  $s$ . The optimum value of  $s$  was

Table 2: Average density estimate, average variance of the mean and average coefficient of variation for the mean distance method on 100, 50 and 25% density levels of the random distribution using 25 replications of sample size 30.

Density Level (%)	Distances Per Point ( <i>s</i> )	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	Average Variance of the Mean	Average Coefficient of Variation (%)
100	1	70,642	2,813,641	26,603,279,495,000	183.3
	2	70,642	295,807	6,744,809,851	27.8
	3	70,642	235,722	1,886,445,916	18.4
	4	70,642	207,981	844,705,527	14.0
	5	70,642	193,203	435,475,897	10.8
	6	70,642	187,140	322,285,948	9.6
	7	70,642	177,934	221,114,382	8.4
	8	70,642	182,906	205,928,119	7.8
	9	70,642	176,190	188,782,443	7.8
	10	70,642	174,352	147,528,450	7.0
50	1	35,321	454,571	118,149,145,990	75.6
	2	35,321	163,961	2,119,511,619	28.1
	3	35,321	115,208	301,273,863	15.1
	4	35,321	103,411	146,876,446	11.7
	5	35,321	100,114	117,295,903	10.8
	6	35,321	95,796	82,007,273	9.5
	7	35,321	95,051	59,227,165	8.1
	8	35,321	94,737	53,315,460	7.7
	9	35,321	92,109	41,623,915	7.0
	10	35,321	93,015	35,810,502	6.4
25	1	17,661	211,039	19,220,959,454	65.7
	2	17,661	78,442	567,246,649	30.4
	3	17,661	58,826	113,531,697	18.1
	4	17,661	50,704	38,863,435	12.3
	5	17,661	47,970	27,890,612	11.0
	6	17,661	47,555	23,512,661	10.2
	7	17,661	46,016	17,061,496	9.0
	8	17,661	46,760	16,726,337	8.7
	9	17,661	44,949	10,517,945	7.2
	10	17,661	45,337	8,560,942	6.5

determined to be the smallest value where the mean distance method overestimated the actual population densities of all three density levels of the random distribution at approximately the same proportions. A large number of distances would render the method ineffective when considering the time required to complete a sample. The optimum value of *s* was determined to be three.

Because the mean distance method overestimated the three random distribution densities at all values of *s* it was inferred that the mean distance method was underestimating the average distance between trees. The mean distance method was measuring less than the actual distance needed, resulting in the overestimation

bias. It was determined that the average distance obtained between trees when *s* equals three represented only 55% of the average distance needed (Tab. 5).

The original program REGEN was altered by dividing the sample point average distance obtained between trees by 0.55, thereby increasing the average physical distance measured between trees and hence the average area occupied per tree, to adjust for the overestimation bias and provide an accurate average density estimate and to test for robustness via a new formula. The adjusted formula is:

$$di = \frac{43560}{0.8661\left(\frac{si}{0.55}\right)^2}$$

Table 4: Average density estimate, average variance of the mean and average coefficient of variation for 4.05 square meter plot sampling on three densities of the random distribution using 500 replications of sample size 30.

Density Level (%)	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	Average Variance of the Mean	Average Coefficient of Variation (%)
100	70,642	70,358	5,796,740	3.4
50	35,321	35,185	2,970,126	4.9
25	17,661	17,791	1,467,529	6.8

Table 6: Average density estimate, overestimation, average variance of the mean and average coefficient of variation for the mean distance method (using the 0.55 adjustment when  $s$  equals three) on the three random distribution densities using 500 replications of sample size 30.

Distances Per Point ( $s$ )	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	Overestimation (%)	Average Variance of the Mean	Average Coefficient of Variation (%)
3	70,642	71,600	1.014	185,406,741	19.0
3	35,321	35,847	1.015	37,010,695	17.0
3	17,661	17,715	1.003	15,394,389	22.1

Table 3: Overestimation factors ([average density estimate – actual density]/actual density) for the mean distance method on three densities of the random distribution using 25 replications of sample size 30.

Distances Per Point ( $s$ )	Overestimation Factor - - - Density Level - - -		
	100%	50%	25%
1	38.83	12.87	11.95
2	4.19	4.64	4.44
3	3.34	3.26	3.33
4	2.94	2.93	2.87
5	2.73	2.83	2.72
6	2.65	2.71	2.69
7	2.52	2.69	2.61
8	2.59	2.68	2.65
9	2.49	2.61	2.55
10	2.47	2.61	2.57

Table 5: Expansion factors (average measured distance/actual distance needed) for the mean distance method on the three random distribution densities using 25 replications of sample size 30 when  $s$  equals three.

Density Level (%)	Average Measured Distance (cm)	Actual Distance Needed (cm)	Expansion Factor
100	22.12	40.41	0.55
50	31.65	57.15	0.55
25	44.30	80.85	0.55

where  $d_i$  = population density estimate per acre at sample point  $i$ ,  $s_i$  = average of the distances measured at sample point  $i$  in feet.

The mean distance method was then tested on the three random distribution densities with 500 replications of sample size 30 when  $s$  equals three (Tab. 6). The results of the tests indicate the overestimation bias was corrected. The mean distance method’s average density estimates for all three random population densities

were within 1.5% of true population density and in close agreement with 4.05 square meter plot sampling results. The results also show that the mean distance method’s average variance of the mean when using the 0.55 adjustment was higher than the average variance of the mean of 4.05 square meter plot sampling for an equivalent density, sample size and number of replications. Average CV’s were relatively high for the mean distance method, ranging from 19 – 22 % where  $s = 3$  (Tab. 6), as compared to 3 – 7% for the 4.05 square meter plot methodology (Tab. 4).

The mean distance method was then tested on all three clumped population densities with 500 replications of sample size 30 to evaluate its robustness across different spatial patterns (Tab. 7). Plot sampling at 4.05

Table 7: Average density estimate, overestimation, average variance of the mean and average coefficient of variation for the mean distance method (using the 0.55 adjustment when  $s$  equals three) on the three clumped distribution densities using 500 replications of sample size 30.

Distances Per Point ( $s$ )	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	Overestimation (%)	Average Variance of the Mean	Average Coefficient of Variation (%)
3	70,617	654,535	9.269	131,109,814,367	55.3
3	35,296	646,187	18.307	4,896,588,697,138	342.4
3	17,636	44,801	2.540	396,354,779	44.4

Table 8: Average density estimate, average variance of the mean and average coefficient of variation for 4.05 square meter plot sampling on the three clumped distribution densities using 500 replications of sample size 30.

Density Level (%)	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	Average Variance of the Mean	Average Coefficient of Variation (%)
100	70,617	67,910	113,729,757	15.7
50	35,296	34,229	29,709,656	15.9
25	17,636	17,179	8,314,850	16.8

square meters was also tested on all three clumped population densities with 500 replications of sample size 30 (Tab. 8). The average density estimates for the mean distance method were not close to the true clumped population densities and the average variance of the mean and average coefficient of variation for the mean distance method at all three clumped population densities when  $s$  equals three was higher than the average variance of 4.05 square meter plot sampling for an equivalent density, sample size, and replication. Even 4.05 square meter plot sampling performed more poorly when seedlings were clumped, with average CV's of 16 – 17% (Tab. 8) as compared with 3 – 7 % for randomly distributed seedlings (Tab. 2).

Ninety-five percent confidence intervals of density estimate for both 4.05 square meter plot sampling and the mean distance method, plus the optimum sample size needed for the mean distance method to achieve 4.05 square meter plot sampling precision, were determined for all three density levels of the random and clumped spatial distributions (Tabs. 9-10). All confidence intervals of the density estimate for 4.05 square meter plot sampling were smaller than the confidence intervals of density estimate for the mean distance method for an equivalent density level and spatial pattern when  $s$  equals three. The sample sizes needed for the mean distance method to achieve 4.05 square meter plot sampling precision at any density level and spatial pattern were at least ten times larger than the sample size of 30 used by a 4.05 square meter plot.

## 5 DISCUSSION

When calculating density at each sample point, the mean distance method's average density estimates were in close agreement with the random distributions, as were the density estimates for 4.05 square meter plot sampling, but the mean distance method was more variable than plot sampling. The mean distance method overestimated actual density and was less precise than 4.05 square meter plot sampling when both methods were tested on the clumped distributions. Since the optimum sample sizes needed for the mean distance method to achieve the same precision as 4.05 square meter plot sampling at all three density levels of the random and clumped spatial distributions were at least 10 times larger than the sample size used by 4.05 square meter plot sampling, it would not be practical to implement, and would prohibit the use of the mean distance method as proposed.

Although the mean distance method looked promising in theory, its lack of success was due to the inclusion of very small distances. Including very small distances, as compared to larger distances, increased and spread out the density estimates considerably which resulted in the mean distance method being less precise than 4.05 square meter plot sampling. Although the mean distance method proved to be accurate at calculating density estimates, the high variability and overestimation bias would preclude its practical application towards quantifying forest regeneration.

Table 9: Density estimate confidence interval (95%) for 4.05 square meter plot sampling on all three densities of the clumped and random distributions using 500 replications of sample size 30.

Spatial Distribution	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	95% Confidence Interval (seedlings/hectare)
Random	70,642	70,358	65,541 - 75,174
Random	35,321	35,185	31,737 - 38,631
Random	17,661	17,791	15,363 - 20,214
Clumped	70,617	67,910	46,579 - 89,239
Clumped	35,296	34,229	23,327 - 45,129
Clumped	17,636	17,179	11,411 - 22,944

Table 10: Density estimate confidence interval (95%) and optimum sample size needed to achieve 4.05 square meter plot sampling precision for the mean distance method (using the 0.55 adjustment when *s* equals three) on all three densities of the clumped and random distributions using 500 replications of sample size 30.

Spatial Distribution	Actual Density (seedlings/hectare)	Average Density Estimate (seedlings/hectare)	95% Confidence Interval (seedlings/hectare)	Optimum Sample Size
Random	70,642	71,600	44,366 - 98,832	960
Random	35,321	35,847	23,680 - 48,014	374
Random	17,661	17,715	9,868 - 25,560	315
Clumped	70,617	654,535	0* - 1,378,717	34,584
Clumped	35,296	646,187	0 # - 5,071,834	4,944,441
Clumped	17,636	44,801	4,982 - 84,617	1,430

\*Actual Value: -69,647

#Actual Value: -3,779,461

Since the overestimation bias can be adjusted by incorporating an overestimation bias constant the mean method shows promise at calculating density in situation such as pole size stand or larger when the average distance between trees would be greater. This would reduce the overestimation bias and decrease the variability of the mean distance method and make it a more likely alternative to traditional milacre plot sampling.

The mean distance method was shown not to be a valid replacement for traditional milacre plot sampling for quantifying forest regeneration due to the close spacing of seedlings on a forest floor. However, the mean distance method did perform fairly well within a random distribution but not within a clustered or clumped population. The mean distance method may be a better choice in rangeland-shrub or dry forest communities where regeneration is less clustered or within large diameter tree conditions where the average distance between trees is greater.

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