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SCHEDULE FUEL TREATMENTS TO FRAGMENT HIGH FIRE HAZARD FUEL PATCHES

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Abstract. Fuel treatment is an important component of wildland fire management. This research revised a mathematical programming model to schedule fuel treatments to fragment fuel patches with high fire intensity hazard. It differs from many previous fuel treatment allocation models that design treatment layouts based on explicitly modeling specific fires' spread within predefined durations. This approach does not rely on the assumption that we can accurately predict future fires conditions. Scheduling fuel treatments to fragment high fire hazard fuel patches has similar effects as scheduling fuel treatments to control fires with long durations. Fuel treatments aimed directly at patch management could effectively lower the risk of future fires that may spread along various directions and with different spread speeds and durations. Tests also suggested that fuel treatment layouts designed to control fires with shorter durations might not perform well when the actual fire duration is much longer. This research presented a new and practical approach in fragmenting fuel patches through efficient fuel treatments allocation.

Keywords: Wildland fire, simulation, optimization, fire spread, patch management.

1 Introduction

Excessive fuels left from long-term fire exclusion increased the risk of high intensity catastrophic wildfires in many forests in the western USA. Fuel treatments can mitigate fire risk (Parisien et al. 2010) by fragmenting fuel patches, slowing fire spread and decreasing fire intensity (Stratton 2004, Fernandes and Botelho 2003). Fuel treatments can also improve the efficiency and safety of future fire suppressions (Agee et al. 2000, Hirsch et al. 2004, Loehle 2004) and lower the chance of fire spreading into wildland urban interface (WUI) (Finney 2001, Mell et al. 2010).

Fuel treatments at different locations often collaborate across a landscape to influence fire spread and intensity (Rytwinski and Crowe 2010). Modeling for fuel treatment locations can help improve the efficiency of fuel reduction programs (Salazar and Gonzalez-Caban 1987, Kaloudis et al. 2005). Many studies have been done to improve the efficiency of spatial fuel treatment layout in mitigating fire risks. Some studies suggested allocating treatments into parallel strips perpendicularly to major fire spread directions to better intercept fire spreads (Fujioka 1985, Catchpole et al. 1989). Others considered fuel treatment allocation as a patch management problem and suggested treatments should be used to fragment high fire hazard patches composed by contiguous and heavy fuels (Agee et al. 2000).

It is often challenging to select fuel treatment locations on a landscape with heterogeneous features such as elevation, fuel types, and values susceptible to potential fire damages. Mathematical models can play important roles to synergize information to assist the selection of landscape fuel treatment locations, or to provide preliminary treatment layout plans that can be further enhanced. Simulation and optimization represent two major types of decision support approaches. Simulation model has the advantage of mimicking detailed fire dynamics by accounting for the changes and influences of fuel, topography and weather. Repeated fire simulations can help evaluate the effectiveness of various fuel treatment layout alternatives. Finney (2006) evaluated the efficiencies of several regular spatial fuel treatment layouts by simulating fire spreads on the treated landscapes. Finney (2008) latterly used a method to search for the fastest fire spread paths by starting and simulating the growth of a row of fires. The result of this study suggests that fuel treatments should be allocated along the fastest fire growth paths to reduce the rates of fire spread. Kim et al. (2009) used a heuristic to compare fire spreads under dispersed, clustered, random, and regularly spaced fuel treatment layouts.

Optimization models have also been developed to schedule fuel treatments to improve our ability to control future fires (Pyne 1984) under various possible fire conditions (He et al. 2004). Hof et al. (2000) developed a linear programming (LP) model to schedule fuel treatments to slow the movement of a specific fire to protect a small number of preselected locations. Wei et al. (2008) developed a mixed integer-programming (MIP) model to schedule fuel treatments to break the fire probability accumulation pathways to lower landscape fire risks for up to two fire weather and duration scenarios. Konoshima et al. (2010) developed a stochastic dynamic programming (DP) approach to combine the decisions of fuel treatment and timber harvesting in a hypothetical landscape to slow the spread of multiple fires. Higgins et al. (2011) developed an MIP model to schedule seasonal resources for prescribed fuel burning. Wei (2012) developed a two-stage model that schedules treatments to create fire control opportunities for a large set of possible future fires. A recent study from Minas et al. (2014) designed an integer programming model to schedule fuel treatments across multiple periods.

Past research suggests that fuel treatments should be used to "fragment high-risk forest landscapes (Acuna et al. 2010)" (e.g., Agee et al. 2000, Finney 2001, Hirsch et al. 2001, Finney 2006, 2008, Wei et al. 2008, Konoshima 2010, Wei 2012). However, most of these models rely on the assumption that we can accurately predict the duration, weather, spread speed, and intensity of future fires, and schedule treatments accordingly. A different modeling principle has also been tested to aim at directly managing high fire hazard fuel patches. A shortest-path network model developed by Bevers et al. (2004) focused on facilitating fuel treatment connectivity. It suggested that a large portion of a landscape needs to be treated to form well-connected fuel breaks to fragment fuel patches. Acuna et al. (2010) developed an iterative approach to integrate forest and fire management under an assumption that timber harvesting could help create non-burnable areas that would help fragment the burnable patches of stands and protect valuable forest from future fires. A study from Contreras et al. (2012) tried to break tree-level fuel connectivity by using a logistic regression model to compare different tree removing options. A model developed by Minas et al. (2014) defined fuel connectivity set and schedule treatments to minimize the number of connected pairs of "old fuel cells" in multiple periods.

This research demonstrated a model revision to fragment high fire hazard fuel patches without explicit predicting fire spread speeds and fire durations. High fire hazard fuel patch is defined here as continuous landscape features that support high intensity fires. This research compared the effectiveness of different fuel treatment layouts in decreasing the potential damages from a large number of fires. Results shows that fuel treatment layouts directly aimed at high fire hazard fuel patch fragmentation can perform consistently well across a broad range of possible future fire situations.

2 Methods

2.1 Review of an existing model Previous research suggested fire spread in a landscape can be modeled by continuously tracing fire spread between adjacent cells through the minimum travel time (MTT) algorithm (Cheng and House 1996, Finney 2002, Sturtevant et al. 2009). Using the MTT algorithm, Wei (2012) developed a MIP model to select fuel treatment locations by explicitly modeling many future fires and their spread with predefined future fire behaviors and durations. This model has been applied in a set of raster landscapes. Fuel treatments were assumed to be able to influence the fire intensity and the rate of fire spread in a treated cell, and consequently alter the spread of future fires. This mathematical formulation is reviewed here.

Set and Subscripts

C and r: the set and index of raster cells in a landscape.

$C^{'}$ and $r^{'}$: the subset and index of raster cells that have high fire intensity.

$C^{''}$ and $r^{''}$: the subset of raster cells that have low fire intensity. Fuel treatment is not required in these cells.

*S and s:*the set and index of raster cells from which ignitions could start.

Q_r *and q:* the set and index of cells directly adjacent to cell r (sharing an edge or a corner).

Parameters

B: the number of cells fuel treatment can be scheduled in.

D: the expected fire duration.

P_s: the probability of a fire igniting from cell s in the next discrete planning period.

L_r: value to be protected from fire in each cell r (loss if burned).

K: a positive constant denoting the delayed fire spreading time by fire control in a treated cell, or in any other cells with low fire intensity.

$\tau_{q,r}$: fire travel time from the center of cell q to the center of cell r without treatment.

Variables

$x_{r'}$: binary variable tracking treatment decisions in cell r'. We assume only cells currently having high fire intensity can be treated.

$x_{r'}$=1 denotes that fuel treatment is scheduled in cell r'; $x_{r'}= 0$ denotes that no fuel treatment is scheduled in cell r'.

$x_{r''}$: denoting cell r'' currently having low fire intensity.

$t_{s,r}$: contiguous variable tracking the fire arrival time to cell r after ignited from cell s.

$y_{s,r}$: binary variable tracking whether fire will burn cell r within a duration D after ignited from cell s; $y_{s,r}$=1 denotes this fire will burn cell r; otherwise $y_{s,r}$=0.

Mathematical formulation

Minimize :

$$Z = \sum_{s \in S} \sum_{r \in C} P_s \times L_r \times y_{s,r} \tag{1}$$

Subject to :

$$t_{s,s} = 0 \qquad \forall\, s \in S \tag{2}$$

$$t_{s,r} \le t_{s,q} + \tau_{q,r} + K \times x_r \quad \forall\, s \in S,\ r \in C,\ q \in Q_r \tag{3}$$

$$y_{s,r} \ge \frac{D - t_{s,r}}{D} \quad \forall\, s \in S,\ r \in C \tag{4}$$

$$\sum_{r' \in C'} x_{r'} \le B \tag{5}$$

$$x_{r''} = 1 \quad \forall\, r'' \in C'' \tag{6}$$

Objective function (1) minimizes the total fire loss from all modeled fires within a predefined duration D. Loss caused by each fire is the total value loss within the fire footprint at the end of duration D. This loss is weighted by the probability of that particular fire ignition within the next discrete planning period (i.e. one year). Equation (2) sets the fire arrival time to cell s as zero when we assume fire is ignited from it. A fire is ignited from every possible ignition cell on the landscape. Equation (3) applies the MTT algorithm to track the earliest time $t_{s,r}$ that fire could arrive the center of cell r from any of its eight adjacent cells q after originated from cell s. The major fire spread direction in each cell represents the fastest fire spread direction (front fire) in that cell. Fires also spread along other directions at slower speeds as flank fires or back fires. It assumes fire spreads in each cell following an elliptical shape (Green et al. 1983) and the value of $\tau_{q,r}$ will be calculated using the major fire spread direction, distances between adjacent cells and the dimension of the ellipse reported by software such as FlamMap (Finney 2006). It assumes cells with low fire intensity could delay fire spread due to the improved fire control efficiency. The amount of time delayed is defined by a parameter K. By setting the value of K larger than the modeled fire duration, it assumes no fire would spread into the center of cell r if the fire intensity in it were low. Equation (4) defines binary variable $y_{s,r}$ working as a switch to track whether fire started from cell s would burn cell r within duration D. If fire reaches the center of cell r within duration D, then $D > t_{s,r}$, therefore $y_{s,r}$ will be set to one by Equation (4); otherwise $y_{s,r}$ could be either zero or one. When given the choice (zero or one for $y_{s,r}$), the model will set it to zero to minimize the fire loss within duration D in objective function (1). Equation (5) is a budget constraint reflecting the number of cells with higher fire intensity to be treated in the landscape. Equation 6) lets the model recognize that any cell with low fire intensity should be considered as same as a treated cell and can be used to delay fire spread. The fuel treatment allocation decision in this model depends on the prediction of many future fires along with their spread directions, spread speeds and durations.

2.2 A revised formulation for fuel patch management An important objective of fuel treatment is to facilitate the future fire control. However, the future fire locations and fire conditions are often difficult to predict due to stochastic weather changes and fire durations. Fuel treatment layout optimized for a specific future fire condition might not provide the best control when the actual condition does not follow what has been planned for. In this research, we will revise the above model to switch emphases from predicting and scheduling treatments to control specific future fires to using treatments to fragment fuel patches. We will compare the effectiveness of different treatment strategies through post-treatment fire simulations.

The above mathematical programming model can be easily revised to focus on high fire hazard fuel patch management. We will use a set of new equations (7) and (8) to substitute the original equations (3) and (4).

$$t_{s,r} \le t_{s,q} + x_r \quad \forall\, s \in S,\ r \in C,\ q \in Q_r \tag{7}$$

$$y_{s,r} + t_{s,r} \ge 1 \quad \forall\, s \in S,\ r \in C \tag{8}$$

This new set of equations eliminates the requirement of predicting and calculating the parameter $\tau_{q,r}$ in Equation (3), which is also highly stochastic and difficult to predict. The revised model will still account for the information such as values to be protected from fire, fire intensities and the fire ignition probability distribution in a landscape. After a fire ignited from a cell s, if cell r is located in the same high fire hazard fuel patch as s, $t_{s,r}$ will be set to zero following the equation (7); otherwise $t_{s,r}$ will be set to one indicating that cells r and s are not in the same high fire hazard fuel patch and fire will not spread from s to r. In equation (8), if cell r will not be burned by the fire ignited in cell s, $y_{s,r}$ will be allowed to be zero after $t_{s,r}$ is set to one; otherwise $y_{s,r}$ will be set to one.

The nature of this revised formulation in fragmenting fuel patches can be described through an example. For demonstration purpose, we first set the probability of fire ignited in every cell to be 0.01 in each year, and set the value to be protected from fire in each cell to be 1.0.

N: the total number of disjointed fuel patches in the landscape after treatment.

i: the index of each fuel patch after treatment.

M_i: the number of cells within each fuel patch i.

With constraint (7), fire started from each of the M_i cells in patch i can spread into all the other cells within the same patch i. An ignition in patch i will cause a fire loss of M_i. If we assume that a cell can be burned repeatedly, the total expected fire loss in each fuel patch i becomes $0.01 \times M_i{}^2$. The total expected future fire loss from the N patches is $0.01 \times \sum_{i=1}^{N} M_i^2$. On a homogeneous landscape with a fixed number of patches (a constant N), the best way to minimize the value of $\sum_{i=1}^{N} M_i^2$ is to evenly distribute the number of cells into all patches (Appendix). In general, if the total area of high fire hazard fuel patches is a constant, fragmenting these areas into a larger number of smaller, isolated and even-sized fuel patches would lead to lower overall landscape fire risk. However, in real world, the probability of fire ignited from each cell s (P_s), and the value to be protected from fires in each cell r (L_r) may vary across a landscape. This model will need to weigh the impacts from these landscape heterogeneities when it fragments the larger fuel patches. The value of N is not a predetermined number either. Instead, it is also the result from fuel treatments.

As we discussed earlier, solutions discovered from this patch management model are not sensitive to the changes of certain fire behaviors such as fire duration, the rate of fire spread and the major fire spread direction. An implicit assumption of this model is that each fire will spread for a very long duration; therefore a fire ignited in a cell will eventually spread into all other cells within the same high fire hazard fuel patch unless it can be stopped at cells of low fire intensities, i.e. fuel breaks. This allows the model to concentrate on a landscape strategy to break fuel patches and ignore certain fire-spread details.

2.3 A method to evaluate the treatment effectiveness Spatial fuel treatments layouts designed under different assumptions are often different. We will evaluate the effectiveness of these layouts by simulating many future fire ignitions and summarize the losses from these hypothetical fires.

1. Define a set of fire scenarios, index by r. For each scenario r, fires are simulated following certain predefined uniformly distributed random fire duration, i.e. 0 to 720 minutes. The major fire spread direction in each cell is determined by a predefined distribution of random fire spread direction, i.e. uniform distribution from 0 to 360 degrees.

2. K fuel treatment layouts, indexed by k, would be tested under each fire scenarios r defined in 1).

3. M replications, indexed by m, are generated to test the effectiveness of each fuel treatment layout k under each specific fire scenario r.

4. For each tested replication m, randomly selected fires are ignited from different locations (cells) on a landscape with their durations and major spread directions randomly fluctuated following the distribution defined by the corresponding fire scenario r in 3). The expected loss of each simulated fire is calculated by multiplying the ignition probability of this fire with the simulated fire loss of this fire.

5. The total expected losses $g_{k,r,m}$, from all simulated fires for fuel treatment layout k will be calculated under each fire scenario r, for each replication m.

The effects of different fuel treatment layouts k in decreasing fire loss under each scenario r are then summarized and compared by using the fire losses calculated from all replications. Tukey's test (see Goldsman and Nelson 1998) is used to compare the mean fire losses of different treatment layout k under each tested fire scenario r. The formulations are listed below.

$$\overline{G}_{k,r} = \frac{\sum_m g_{k,r,m}}{M} \quad \forall\ k, r \tag{9}$$

$$S_r = \sqrt{\frac{\sum_k \sum_m \left(g_{k,r,m} - \overline{G}_{k,r}\right)^2}{(M-1) \times K}} \quad \forall\ r \tag{10}$$

Equation (9) calculates $\overline{G}_{k,r}$, the average landscape fire losses across all M replications used to study the effectiveness of the fuel treatment layout k under the tested fire scenario r. S_r calculated by equation (10) is the standard deviation of all $M\times K$ replications tested across K fuel treatment layouts. Tukey's confidence intervals for $u_{k,r}-u_{k',r}$ (difference between the true means of treatment k and k' under fire scenario r) are estimated by equation (11) as

$$\overline{G}_{k,r} - \overline{G}_{k',r} \pm \frac{Q_{K,v}^{(\partial)}}{\sqrt{2}} \times s \times \sqrt{\frac{2}{M}} \quad \forall\, r \qquad (11)$$

for all different k and k', where $Q_{k,v}^{(\partial)}$ is the 1- α (for example $\alpha = 0.05$) quintile of the Studentized range distribution with parameter K and $v = K \times (M-1)$. We will compare whether the difference between any two expected fire losses due to different treatment layouts is significant under the confidence coefficient α for each tested fire scenario r.

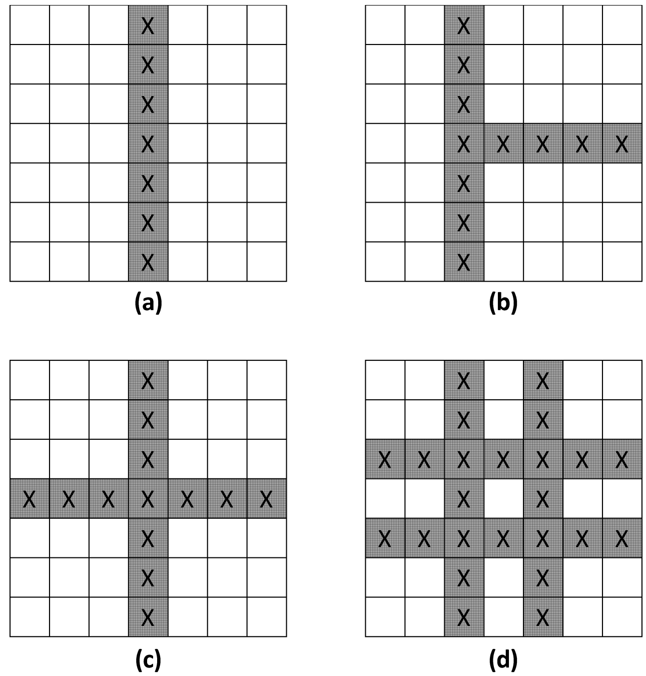

Figure 1: Four fuel treatment levels (seven-cell, eleven-cell, thirteen-cell, and twenty-four-cell) in an artificial landscape of 7×7 cells using the patch management model. These tests assume future fires cannot spread across treated cells with sufficient resources available for suppressing low intensity fires.

3 Results

3.1 Hypothetical test case We first tested the revised model to schedule treatments in an artificial landscape with 7×7 cells. Four levels of fuel treatment areas (seven-cell, eleven-cell, thirteen-cell, and twenty-four -cell) were modeled to fragment high fire hazard fuel patches. These tests also assume there would be sufficient suppression resources to stop future fires within treated cells. Allocating treatments into seven, eleven, thirteen and twenty-four cells each breaks the landscape into two, three, four or nine disjointed smaller fuel patches regardless of the specific rate of fire spread and major fire spread direction in each cell (Figure 1).

3.2 Realistic test case A small portion of Sequoia and Kings Canyon National Parks (SEKI), with an extent of 3.6 by 3.6 km is used as another test example here. This landscape is rasterized into four hundred 180m wide raster cells. We first ran the original fire spread based mathematical programming model (Wei 2012) by assuming a prevailing southwest wind at eight km per hour with moderate understory fuel moisture condition. FlamMap is used to quantify fire behavior in each cell including the major fire spread direction, the fire flame length, the dimension of the burn ellipse and the rate of fire spread. For comparison, we also run the patch management model to focus directly on high fire hazard fuel patch fragmentation. This new model does not require us to retrieve certain fire behavior data such as spread speed and spread directions from FlamMap, and it does not require an assumption of future fire duration. In both tests, a 2.44 m (eight feet) flame length threshold is used as an example to separate cells with high or low fire intensity potentials. We assume only cells with a predicted high fire flame length greater or equal than 2.44m can benefit from fuel treatment, which will decrease the flame length in treated cells to be lower than the 2.44m threshold.

The value to be protected from fire in each cell is assumed to depend on the presence of WUI (with a value of 1.0 per cell) and other forests (0.4) within that cell. These values vary between locations across the study site (Figure 2a). The maximum value that a cell can have is 1.4 (Figure 2a). The annual ignition probability P_s assigned to each potential fire ignition cell s (Figure 2b) is calculated based on the historical ignition frequency in each cell divided by the total number of years within which we tracked the fire ignitions in SEKI. In this test case, we used all the ignitions during the past 83 years.

3.3 Treatments comparison We compare the three 8-cell fuel treatment solutions (described in Figure 3) corresponding to three tested fire duration assumptions: 1) patch management, which is equivalent to assuming infinite fire duration, 2) a shorter 360-minute fire duration, and 3) a longer 24-hour fire duration. Scheduling

Table 1: The distributions of high fire hazard fuel patches with different fuel treatment layouts configured based on various fire duration assumptions.

Seq. No. of hazard fuel patches sorted by patch area	Patch size without treatment (ha)	Patch size after treated 8-cell under various fire duration assumptions		
		360-min duration	24-hr duration	Patch management Inf duration
1	599	573	288	288
2	10	10	156	146
3	6	6	97	97
4	3	3	16	26
5	3	3	10	10
6	3	3	10	10
7	3	3	6	6
8	3	3	6	6
9			3	3
10			3	3
11			3	3
12			3	3
13			3	3

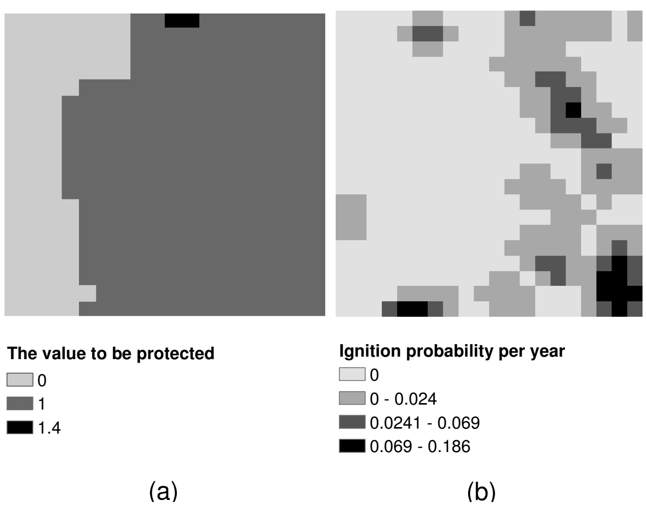

Figure 2: (a) The value to be protected from fire in each cell is assumed to depend on the presence of WUI (with a value of one per cell) and other forests (0.4) within that cell. (b) The annual probability of fire ignited from each cell is calculated using historical records.

fuel treatments under the assumption 1) only requires data to identify cells that potentially support high fire intensities. Selecting treatment locations under either the assumption 2) or 3) requires more detailed fire behavior data including both fire intensity and the rate of fire spread along different directions in each cell.

Without fuel treatment, there are currently eight high fire hazard fuel patches composed by cells with high fire intensity potentials. The size distribution of these patches is described in Table 1. After scheduling treatments in eight cells, the 20×20 landscape is fragmented into many smaller and disconnected high fire hazard fuel patches.

Implementing the explicit patch management method is equivalent to modeling fires with infinite duration. Under this case, the model suggests an optimal fuel treatment layout as shown by Figure 3a. In comparison, with the assumption of a shorter fire duration (D is set to 360 minutes in Equation (4)), the model designs the optimal eight-cell fuel treatment layout as shown in Figure 3b. For planning scenarios with a longer (although not infinite) fire duration (D is set to 24-hr in Equation (4)), the optimal fuel treatments locations are given in Figure 3c.

A direct observation of the distributions of high hazard fuel patches reveals some differences of optimal spatial fuel treatment patterns under different fire duration assumptions. The largest high fire hazard fuel patch is 599 hectares in size without treatment. In the case of using the 360-minute fire duration assumption, after eight cells are treated, the landscape still has eight high fire hazard fuel patches. However, the largest one is twenty-six hectares smaller than the largest high fire hazard fuel patch before treatment. When scheduling treatments under the assumption of 24-hour fire duration, the number of high fire hazard fuel patches increases to thirteen, and the size of the biggest patch decreases to about half of the largest patch before treatment. Treatments help break the landscape into a larger number of high fire hazard fuel patches under longer fire duration assumption. By implementing the patch fragmentation strategy with

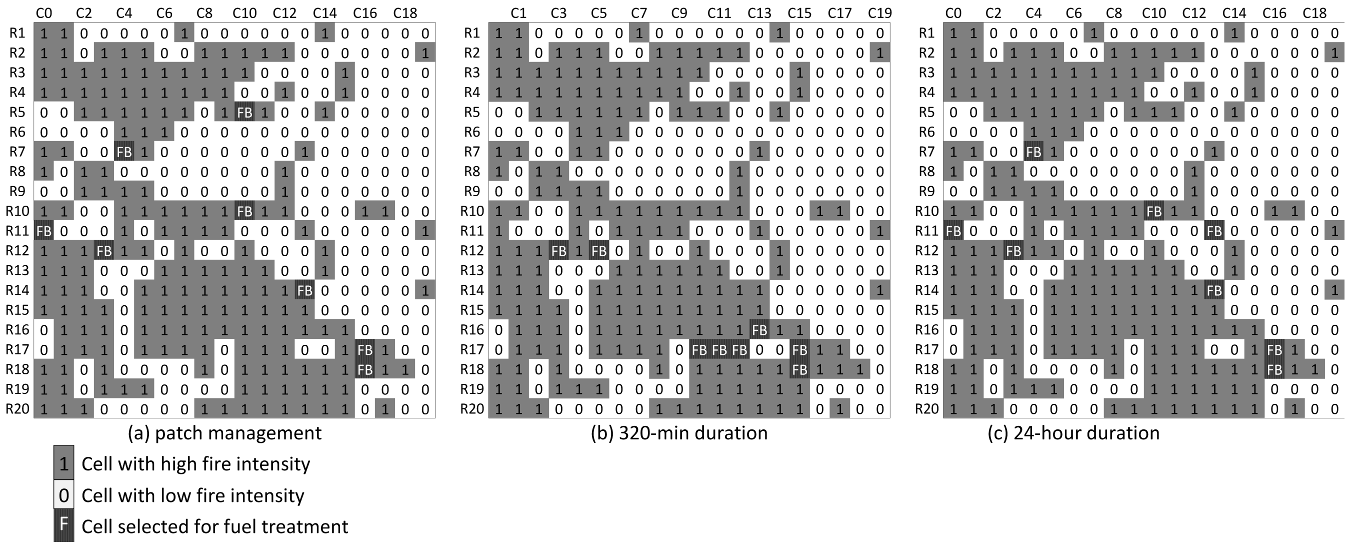

Figure 3: Spatial fuel treatment patterns and the high fire hazard fuel patches after treatments under three modeled fire duration assumptions.

Table 2: Fires are simulated from every possible ignition locations on the tested landscape under a set of preselected fire scenarios. This table compares the efficiency of different fuel treatment layouts in lowering the landscape expected fire losses under each preselected future fire scenarios. The simulation duration of each fire is assigned based on a random draw from a corresponding uniform distribution. The major fire spread direction in each raster cell is also determined by a random draw from the uniform distribution with a range between 0 and 360 degrees.

Stochastic Scenario Seq.	Simulation duration used for post treatment evaluation (minutes)		Expected fire loss after fuel treatments scheduled under different assumptions			Comparing fuel treatment effectiveness using the Tukey's method (95% confidence) #
	From	To	360-min	24-hr	Patch	
1	0	360	18.0	18.2	18.1	No solution is better
2	0	720	33.8	25.0	25.0	24hr>360m,Patch>360m
3	0	1,800	88.6	31.0	30.2	24hr>360m,Patch>360m
4	0	3,600	144.0	33.4	32.7	24hr>360m,Patch>360m
5	0	7,200	178.2	34.8	33.8	24hr>360m,Patch>360m
6	0	14,400	195.4	35.4	34.4	24hr>360m,Patch>360m

#In the last column, A > B means solution based on duration A is significantly better than B.

the assumption of infinite fire duration, this model will still schedule fuel treatments to fragment the landscape into thirteen patches with the largest patch of 288 ha and the second largest patch of 146 ha. The patch distribution between using the 24-hour duration and using the infinite fire duration are very similar (Table 1).

3.4 Comparing the treatment effectiveness The three fuel treatment solutions (corresponding to the assumed 360-minute fire duration, 24-hour fire duration, and the infinite fire duration/patch managements) are tested under each of the six random fire scenarios. In each scenario, we simulate fires ignited from all possible locations in a landscape under random fire durations range from (0, 180) minutes up to (0, 14,400) minutes (Table 2) depending on the specific scenario tested. The major fire spread direction in each cell also randomly varies between zero and 360 degrees. The Tukey's test is used to compare whether there are significant differences between the effects of different fuel treatment layouts in lowering the total expected landscape fire losses.

According to the test results (Table 2), if all simulated fires last randomly between 0 and 360 minutes, none of the fuel treatment layouts would perform significantly better than any of the other two fuel treatment layouts. However, fuel breaks scheduled for fires lasting for 24-hr or infinite are significantly more effective in controlling long duration fires. Based on the testing results, fuel treatment layouts aimed at patch management or at controlling fires of 24-hr duration still perform well when fires have short duration. However, the treatment layout aimed at controlling fires with shorter durations does not perform as well when the durations of future fires are much longer.

4 Discussion and conclusion

This research provides a revised mathematical formulation to directly break contiguous high fire hazard fuel patches. By assuming infinite fire duration, it also assumes that any fire ignited within a high fire hazard fuel patch would eventually burn the entire patch. However, a fire could not spread across treated areas as we assumed that suppression would be effective in stopping low intensity fires. This patch management strategy can also account for the spatial distributions of fire ignitions and the values to be protected from fires.

Studies in the past show there are often "dramatic" variations in future fire conditions (Boychuk and Martell 1996). Because of the uncertainty in future fire ignition locations, fire weathers (wind, moisture etc.), fuel accumulation speed, and fire suppression conditions, finding one fuel treatment layout that can perfectly fit to all future fire scenarios is difficult, if not impossible. Some studies pointed out that fuel continuity is an important factor influencing fire risk (Arkle et al., 2012; Ireland et al., 2012; Contreras et al. 2012). Instead of predicting and integrating a large set of detailed future fire scenarios into a fuel treatment layout model, we used an optimization model to directly fragment high fire hazard fuel patches.

Computing time could become a limit when a large set of stochastic fire scenarios are required to be modeled to inform the landscape fuel treatment decisions. The patch management model introduced here is a deterministic model, or can be considered as a model built on the worst case fire scenario (fires with infinite duration). It avoids the explicitly modeling of a large number of highly stochastic future fire scenarios. In addition, because the patch based fuel treatment design does not depend on the accurate prediction of fine scale stochastic fire events, it would be less demanding on creating and using high quality fire samples through accurate fire simulations or fire surveys.

Although the patch management model saves us from modeling multiple future fire scenarios, we still need to model fuel treatments by considering the possible fire ignition patterns. To assign a set of relative and fair weights to all modeled fire ignitions, we chosen to weight the potential loss of each fire by its ignition probability in the current model. We also allow each cell to be burned for multiple times under the assumption that each ignition would be independent. A more complex but potentially more realistic assumption would be allowing a fire to burn each cell only once during certain time interval (i.e. a planning period). However, under this assumption, we will need to explicitly model the sequences of different fire occurrences, and track the interactions between fires, i.e. how earlier fires would influence the spreads of later fires. This would be an interesting future study, although it would also be much complicated.

Patch management not only can be used in breaking large patches of high fire hazard fuels, but also may be implemented in preventing the spread of other detrimental disturbance agents such as insects and diseases, or invasive species etc. Patch management is a challenging landscape level decision problem due to the possible spatial variations of size, shape, connectivity and location of patches. This research introduced and tested a modeling method to fragment patches in fuel management, which also has the potential of being used to model the spread of insects and decease, or to control the spread of invasive species. This also represents an interesting future study.

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A Appendix

An example of how to best allocate β cells into N disjointed patches to minimize cell interactions in case of fire spreading between cells in each patch.

Minimize

$$M_1^2 + M_2^2 + M_3^2 + \cdots + M_N^2$$

St:

$$M_1 + M_2 + M_3 + \cdots + M_N = \beta \tag{12}$$

$$M_1, M_2, M_3 \ldots\ldots M_3 \geq 0$$

This problem can be solved through Lagrangian method. We can use v to denote the shadow price of equation (12):

$$\begin{aligned} \mathrm{L}\,(M_i, \mathrm{v}) = & M_1^2 + M_2^2 + M_3^2 + \cdots + M_N^2 + \\ & v \times (\beta - M_1 - M_2 - M_3 - \cdots - M_N) \end{aligned} \tag{13}$$

At the stationary point, both $\frac{\partial L(M_i,\mathrm{v})}{\partial M_i} = 0$, and $\frac{\partial L(M_i,\mathrm{v})}{\partial v} = 0$, and therefore:

$$M_1 = M_2 = M_3 \cdots = M_N$$