ISSN 1946-7664. MCFNS 2021
Vol. 13, Issue 2, pp. 14–26
Mathematical and Computational Forestry & Natural-Resource Sciences

AVAILABLE ONLINE AT HTTP://MCFNS.COM

Submitted: Feb. 2, 2021
Accepted: Oct. 22, 2021
Published: Oct. 30, 2021
Last Correction: Nov. 6, 2021

ALTERNATE RANKED SET SAMPLING FOR SKEWED AND MOUND SHAPED SYMMETRIC DISTRIBUTIONS: ACCOUNTING FOR FORESTRY AND ENVIRONMENTAL RESEARCH

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Abstract. Ranked Set Sampling (RSS) is a sampling strategy which is advantageous when measurement of sampling units is very difficult but when small sets of units can be ranked according to other methods that do not require actual measurements. The units corresponding to each rank are used in RSS and RSS performs better than simple random sampling (SRS) when estimating the population mean of forestry or environmental parameters (say, below ground biomass). A new RSS procedure based on alternate order statistics for estimating the population mean (ARSS) is suggested in this paper. ARSS measures only the first, third, fifth and so on units so that the information on remaining order statistics is represented by their respective neighboring order statistics. The bias correction term in the proposed estimator is included and calculated for some skewed and symmetric (both mound and U shaped) distributions. The estimators under ARSS are then compared to the estimators based on balanced RSS and Neyman's optimal unbalanced RSS allocations. Based on the computed Relative Precisions (RPs), estimators based on ARSS are recommended for even set sizes of skewed distributions and odd set sizes of mound shaped symmetric distributions. RPs of these distributions are uniformly better than the other two methods (balanced and Neyman's RSS). To demonstrate the performance of the different estimators, an example from forestry that estimates total biomass of three tree species is presented. The proposed method is efficient in forestry and environmental applications.

Keywords: above ground biomass, alternate ranked set sampling, distributions, ranked set sampling, relative precision, unbiasedness

1 Introduction

In probabilistic sampling, there is a wide variety of methods available including simple random sampling (SRS), systematic sampling, variable probability sampling, and multistage/multiphase designs utilizing auxiliary information from aerial photography, satellite imagery, or other sources (Kershaw et al., 2016; Yang et al., 2019). The recommended sampling method often depends upon the situation. For example, point sampling for large-scale timber surveys, distance sampling for capturing animal abundance, probability proportion to prediction sampling (3P sampling) for stand-level inventory, and sampling sparse populations using auxiliary variables. Two recent developments within environmental sampling (Barnett, 1999; Chandra et al., 2020; Latpate et al., 2021) are ranked set sampling (RSS) after McIntyre (1952) and adaptive cluster sampling (ACS) after Thompson (1990) and are being used in a variety of forestry and environmental applications (Acharya et al., 2000; Chandra et al., 2011, 2019; Martin et al., 1980). RSS was developed as a sampling method to improve precision of the estimator of the mean in situations where the actual measurement of the attribute of interest is difficult and costly but the ranking of sampling units by alternative methods that do not require actual measurements is easy. In forest inventory and research, such situations arise frequently in many field surveys. For example, measurement of parameters like canopy or wood volume of a tree or forest stand are rather difficult; however, ranking trees or stands by visual inspection in comparison with the other neighboring trees or stands in the same population is relatively easy. There are a diverse number of applications in forestry where this concept can improve sampling efficiency. For example, intensity of insect pest attacks, biodiversity assessment, assessment of above-or below-ground biomass, pollution impacts, and so on. RSS has been applied in several forestry field surveys. For example, Halls and Dell (1966) used RSS for estimating weights of browse and herbage in a pine-hardwood forest of East Texas. Evans (1967) employed this approach in regeneration surveys for long-leaf pine trees. Martin et al. (1980) used RSS to estimate shrub phytomass in Appalachian oak forests. Cobby et al. (1985) utilized RSS in the investigation of grass and grass-clover swards. Nelson et al. (1987) used RSS to study nutrition status of *Populus deltoides* plantations in the lower Mississippi River Valley. Mode et al. (1999) used RSS to assess salmon production across stream habitat areas in the Pacific Northwest and provided specific guidelines for situations where RSS is appropriate and cost effective for ecological and environmental field sampling. Kvam (2003) applied RSS to binary water quality data with covariates. Tarr et al. (2005) incorporated RSS in their attempt to map accuracy of soil variables using soil electrical conductivity as a covariate. Platt et al. (1988) used RSS to assess the population dynamics of old-growth long-leaf pine (*Pinus palustris*). Wang et al. (2009) showed how RSS can be used to increase efficiency and reduce costs in fishery research. Chandra et al. (2019) suggested using RSS to estimate response of developmental programs (including *Joint Forest Management Programmes* of the Government of India). More recently, Kumar et al. (2019) investigated the use of RSS in assessing bark eating caterpillars, *Indarbela quadrinotata* (Walker), in *Populus deltoides* plantations in Western Uttar Pradesh and Uttarakhand, India.

1.1 Ranked Set Sampling, Estimators and Relative Precisions

The original version of McIntyre's (1952) RSS approach was defined as follows: An SRS of size k is selected from the population and the k sampling units are ranked based on personal judgment or a concomitant variable, X, without any actual measurement. Then the unit with rank 1 is identified and selected for actual measurement and the remaining units of the initial sample are discarded. Next, a second SRS of size k is drawn, the units of this sample are ranked using the same criteria as in sample one, and the unit with rank 2 is selected for measurement and the remaining units are discarded. This process is continued until an SRS of size k is selected and ranked and the unit with rank k is selected for measurement (i.e., k SRSs of size k are selected and the 1 to k ranked observations selected for measurement). This whole process is referred to as a cycle. The cycle is then repeated m times to get a ranked set sample of size $n = km$. This procedure is termed a balanced RSS or RSS with equal allocation. An illustration of a ranked set sample of trees for estimating any parameter through the concomitant variable height using the set size two and three with number of cycles as three and two is shown in Fig. 1.

Let $Y_{((i:k)j)}$,$i = 1, 2 \ldots\ k$; $j = 1, 2 \ldots, m$, denote the measured value of the characteristic under study of the ith rank order in the jth cycle. All the mk measured units out of total k^2m selected units under balanced RSS are demonstrated as:

$$\begin{array}{cccc} Y_{(1:k)1} & Y_{(2:k)1} & \cdots & Y_{(k:k)1} \\ Y_{(1:k)2} & Y_{(2:k)2} & \cdots & Y_{(k:k)2} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{(1:k)m} & Y_{(2:k)m} & \cdots & Y_{(k:k)m} \end{array}$$

Here it is noted that, for fixed i, the $Y_{(i:k)j}$, $j = 1, 2, \ldots, m$, are independently and identically distributed with mean and variance, $\mu_{(i:k)}$ and $\sigma^2_{(i:k)}$ respectively. The population values for $\mu_{(i:k)}$ and $\sigma^2_{(i:k)}$ are documented in literature for well-known distributions (for sufficiently large k, see (Harter and Balakrishnan, 1996; Hastings et al., 1947; Sarhan and Greenberg, 1962)). Suppose the population mean and variance is denoted by μ and σ^2 which are assumed to be unknown. Under balanced RSS, an unbiased estimate of μ (McIntyre, 1952) is the simple average of all measurements:

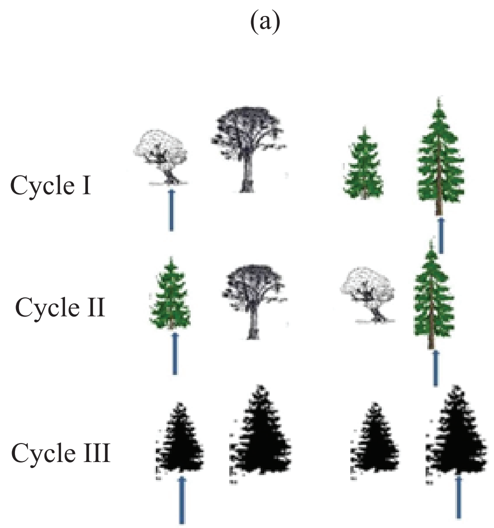
$$\overline{Y}_{(k)bal} = \frac{1}{km}\sum_{i=1}^{k}\sum_{j=1}^{m} Y_{(i:k)j}$$

where, the subscript '*bal*' represents a balanced RSS. The variance of $\overline{Y}_{(k)bal}$ is:

$$\begin{aligned} Var\left(\overline{Y}_{(k)bal}\right) &= \frac{1}{k^2m^2}\sum_{i=1}^{k}\sum_{j=1}^{m} Var\left(Y_{(i:k)j}\right) \\ &= \frac{1}{k^2m^2}\sum_{i=1}^{k}\sum_{j=1}^{m}\sigma^2_{(i:k)} \end{aligned}$$

or

$$Var\left(\overline{Y}_{(k)bal}\right) = \frac{1}{k^2m}\sum_{i=1}^{k}\sigma^2_{(i:k)}$$



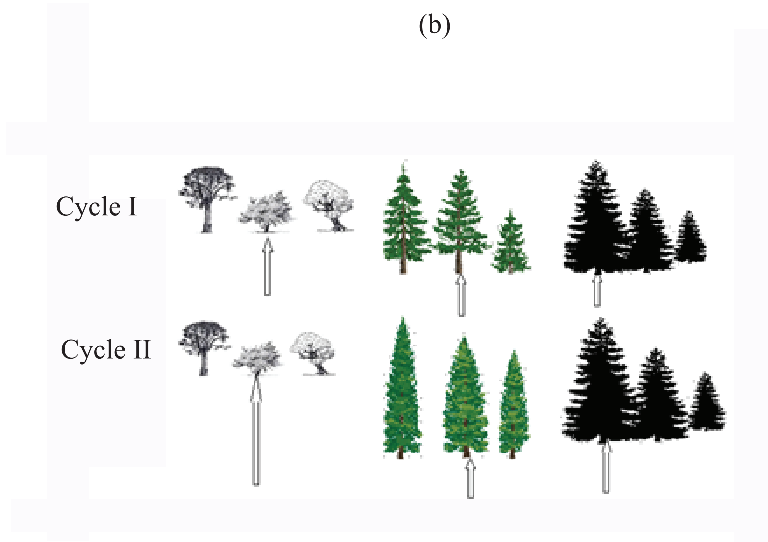


Figure 1: Illustration of a balanced Ranked Set Sample (RSS) for selection of trees with (a) $k = 2$ and $m = 3$, and (b) $k = 3$ and $m = 2$. (arrows mark trees selected for measurement, based on rank of height)

In an unbalanced RSS or RSS with unequal allocation, not all rank order sample units are measured an equal number of times. Suppose $m_i \geq 1$ measurements are made corresponding to the i^{th} rank, $i = 1, 2, \ldots, k$ giving a total of $n = \sum_{i=1}^{k} m_i$ actual measurements for the RSS sample. The unbalanced RSS is displayed as:

$$\begin{array}{cccc} Y_{(1:k)1} & Y_{(2:k)1} & \cdots & Y_{(k:k)1} \\ Y_{(1:k)2} & Y_{(2:k)2} & \cdots & Y_{(k:k)2} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{(1:k)m_1} & Y_{(2:k)m_2} & \cdots & Y_{(k:k)m_k} \end{array}$$

where, $Y_{(i:k)j}$, denotes the measured unit of the i^{th} ordered sample unit during the j^{th} cycle $(i = 1, 2, \ldots, k; j = 1, 2, \ldots, m_i)$.

In unbalanced RSS, the unbiased estimator of μ is:

$$\overline{Y}_{(k)ubal} = \frac{1}{k}\sum_{i=1}^{k}\frac{T_i}{m_i}$$

where, $T_i = \sum_{j=1}^{m_i} Y_{(i:k)j}$, and the subscript '*ubal*' represents an unbalanced RSS. The variance of $\overline{Y}_{(k)ubal}$ is:

$$\begin{aligned} Var\left(\overline{Y}_{(k)ubal}\right) &= \frac{1}{k^2}\sum_{i=1}^{k}\sum_{j=1}^{m_i}\frac{Var\left(Y_{(i:k)j}\right)}{m_i^2} \\ &= \frac{1}{k^2}\sum_{i=1}^{k}\sum_{j=1}^{m_i}\frac{\sigma_{(i:k)}^2}{m_i^2} \end{aligned}$$

or

$$Var\left(\overline{Y}_{(k)ubal}\right) = \frac{1}{k^2}\sum_{i=1}^{k}\frac{\sigma_{(i:k)}^2}{m_i} \tag{1}$$

The optimal allocation for RSS is like Neyman's optimal allocation (Thompson, 2012) in stratified random sampling, and is given by:

$$m_i = \frac{n\sigma_{(i:k)}}{\sum_{i=1}^{k}\sigma_{(i:k)}} \tag{2}$$

The corresponding variance is obtained by using (2) in (1) becomes:

$$Var\left(\overline{Y}_{(k)opt}\right) = \frac{1}{nk^2}\left(\sum_{i=1}^{k}\sigma_{(i:k)}\right)^2 = \frac{\overline{\sigma}^2}{n}$$

where, $(1/k)\sum_{i=1}^{k}\sigma_{(i:k)}$ denotes the average of the within-rank standard deviations.

1.2 Broader Comparison of RSS with 3P Sampling

In forest surveys we often come across situations where actual measurement of sampling units is very difficult but the ranking of the units or measuring auxiliary variable is very easy. In probability proportional to prediction (3P) sampling (Grosenbaugh, 1963, 1965), one measures or predicts (visually or through some other simple tool) the auxiliary variable which is strongly positively correlated with the variable of interest. The higher the degree of correlation, the more efficient the 3P sampling technique becomes. After identification of the auxiliary variable using a preliminary/pilot survey or prior knowledge of the forest population, 3P sampling proceeds in two phases. In the first phase, a set of sampling units is selected from the population using conventional sampling like SRS or systematic sampling. In the second phase, a 3P subsample of the first phase sample the sampling units is selected with unequal probability of selection as follows:

First, identify the range $\left[x_{(l)}, x_{(u)}\right]$ of values of the auxiliary variable in the entire population very carefully because a sampling unit encountered with an auxiliary value outside the range n will be selected with probability of 1 and can lead to inefficiencies and potential biases if the number of such units is large. Then, measure or estimate the auxiliary value of the sampling unit selected in the first phase, suppose it is $x_{(m)}$. A random value $x_{(r)}$ is then selected from $\left[x_{(l)}, x_{(u)}\right]$. If, for a sampling unit, $x_{(m)} \geq x_{(r)}$, the unit is selected for the final sample for estimation purposes. In this case the more difficult and costly to measure variable of interest is measured on that sampling unit. If $x_{(m)} < x_{(r)}$, the sampling unit is discarded and does not become part of the second-phase sample and the surveyor simply moves on to the next sampling unit in the first-phase sample. This rule simply makes probability of inclusion proportional to predicted value and forces some of largest sampling units to be measured.

In a general comparison between 3P sampling and RSS, it is observed that 3P sampling involves measuring or estimating the auxiliary variable quantitatively. The quantitative values of the auxiliary variable are required to determine whether sampling units selected in the first phase are finally selected for actual measurement or not in the second phase. Measurement of auxiliary variables requires substantial costs even though the costing is cheaper. While in the case of RSS, instead of giving quantitative values, only comparative ranks of those units selected in the first phase are required. This process of RSS reduces costs, and the benefits of (i) stratified random sampling is obtained in which final sample is comprised of units from each rank order, similar to (ii) systematic ordered list sampling. Secondly, an auxiliary variable which is strongly positively correlated with the study variable can be difficult to obtain in some cases. In comparison, RSS does not identify the nature of auxiliary variable. Instead, it only ranks the sampling units by visual inspection. Both 3P sampling and RSS do not require two or more visits to the field. Simultaneously, with the ranking process, one immediately decides whether the selected unit is to be measured or not. RSS reduces the sample size required for actual measurement to a larger extent than 3P without compromising the efficiency of the estimator. In balanced RSS, out of the mk^2 sampling units, one measures only mk sampling units. This is one of the important requirements for efficient forest surveys (Iles, 2003, 2012; Kershaw et al., 2016). With this in mind, the use of RSS is recommended as a potentially beneficial sample design for forest related studies. The proposed method of alternate ranked set sampling (ARSS), described in Section 2, further improves the precision over balanced and unbalanced RSS with an additional reduction in sample sizes to half in comparison of balanced RSS.

2 ALTERNATE RANKED SET SAMPLING

The Alternate RSS (ARSS) procedure is proposed to improve efficiency of forestry field surveys that require smaller sample sizes of actual measurements. In ARSS, we measure only the alternate order statistic (i.e., the first, third, fifth and so on) so that the information for the remaining ordered sample units are represented by their respective neighboring ordered sample units. For example, the information for the second ordered sampling unit is represented by its neighboring first and third order sample units and so forth. The ARSS procedure is a method to reduce sample size of actual measurements to half without compromising on the precision of the estimators. The proposed method of ARSS is explained as follows:

Select a simple random sample of size k from the population, rank them as described above for RSS and choose the 1st ordered sample unit for actual measurement. Select another random sample of size k, rank them as before and select the 3^{rd} ordered sample unit for actual measurement. This process is continued until the k^{th} (when k is odd) or $(k-1)^{th}$ (when k is even) ordered sample units are selected for actual measurements.

It should be noted that there are M ordered sample units for actual measurements, where:

$$M = \begin{pmatrix} \frac{k}{2},\, if\, k\, is\, even \\ \\ \frac{k+1}{2},\, if\, k\, is\, odd \end{pmatrix}$$

and represents the median of the set size taken for the study. This whole process is repeated r times to get the balanced RSS of size $n = rM$. If $r = 2$, M becomes set size for even k, as taken for the original RSS approach.

Under the balanced ARSS (or ARSS with equal allocation) model, an estimator of μ is proposed to be:

$$\overline{Y}_{(k)abal} = \frac{1}{rM}\sum_{i=1}^{M}\sum_{j=1}^{r} Y_{(2i-1:k)j} + C_k$$

The subscript '*abal*' denotes ARSS with balanced RSS and C_k is a correction factor to be determined depending upon the set size k. The estimator $\overline{Y}_{(k)abal}$ will be unbiased if C_k is chosen to be:

$$C_k = \mu - \frac{1}{M}\sum_{i=1}^{M}\mu_{(2i-1:k)} \tag{3}$$

The value of C_k may or may not be known in advance. If the nature of the distribution under study is known in advance, the corresponding C_k is also known and is calculated using Eq. (3). In section 3, we first attempt ARSS with a few known distributions (Section 3.1) to calculate the values of C_k. However, in cases where the nature of the distribution is not known in advance, first, C_k must be estimated followed by estimating the population mean and corresponding Mean Square Error (MSE). This case is demonstrated in Section 3.2 using a dataset from a sample survey in a forest as an example.

The MSE of $\overline{Y}_{(k)abal}$ is given by:

$$\begin{aligned} MSE\left(\overline{Y}_{(k)abal}\right) &= \frac{1}{a^2M^2}\sum_{i=1}^{M}\sum_{j=1}^{a} Var\left(Y_{(2i-1:k)j}\right) + b^2 \\ &= \frac{1}{aM^2}\sum_{i=1}^{M}\sigma^2_{(2i-1:k)} + b^2 \end{aligned}$$

where:
$b = Bias\left(\overline{Y}_{(k)abal}\right) = \mu - \Big((1/M\ )\sum_{i=1}^{M}\mu_{(i:k)} + C_k\Big)$.

To compare the performances of the three methods (balanced RSS, ARSS and Neyman's optimum) the following three relative precision (RP) values with respect to SRS with $n = aM$ measurements are computed:

$$RP_1 = \frac{Var\left(\bar{Y}_{SRS}\right)}{Var\left(\bar{Y}_{(k)bal}\right)} = \frac{\sigma^2}{\frac{1}{k}\sum\limits_{i=1}^{k}\sigma^2_{(i:k)}} = \frac{\sigma^2}{\overline{\overline{\sigma^2}}} \tag{4}$$

$$RP_2 = \frac{Var\left(\bar{Y}_{SRS}\right)}{Var\left(\bar{Y}_{(k)abal}\right)} = \frac{\sigma^2}{\frac{1}{M}\sum\limits_{i=1}^{k}\sigma^2_{(2i-1:k)} + aMb^2} \tag{5}$$

$$RP_3 = \frac{Var\left(\bar{Y}_{SRS}\right)}{Var\left(\bar{Y}_{(k)opt}\right)} = \frac{\sigma^2}{\bar{\sigma}^2} \tag{6}$$

where $\overline{\sigma^2} = (1/k\ )\sum_{i=1}^{k}\sigma^2_{(i:k)}$ is the average of the within-rank variances.

Additionally, the RP of ARSS compared to balanced RSS (RP_4) is derived theoretically. For this purpose, we equate $n = mk = aM$ which gives $a = (mk/M)$. For even k, $a = 2m$. The derived value of RP_4 is then given by:

$$\begin{aligned} RP_4 &= \frac{MSE\left(\bar{Y}_{(k)bal}\right)}{Var\left(\bar{Y}_{(k)abal}\right)} \\ &= \frac{M}{k}\frac{\sum\limits_{i=1}^{k}\sigma^2_{(i:k)}}{\left(\sum\limits_{i=1}^{M}\sigma^2_{(2i-1:k)} + mkMb^2\right)} \end{aligned} \tag{7}$$

Similarly, RP of ARSS with respect to Neyman's optimum (RP_5) is given by:

$$\begin{aligned} RP_5 &= \frac{Var\left(\overline{Y}_{(k)opt}\right)}{MSE\left(\overline{Y}_{(k)abal}\right)} \\ &= \frac{M\overline{\sigma}^2}{\left(\sum\limits_{i=1}^{M}\sigma^2_{(2i-1:k)} + aM^2b^2\right)} \end{aligned} \tag{8}$$

It is known that the RP of Neyman's allocation for positively skewed distributions is substantial over other allocation models which are based upon each order statistics. Some available allocation models for skewed distributions are balanced RSS (McIntyre, 1952), the 't' and '$(s,\ t)$' models (Kaur et al., 1997), systematic allocation model (Tiwari and Chandra, 2011) and simple allocation models (Bhoj and Chandra, 2019; Chandra et al., 2018). However, when the estimator is based on symmetric distributions, the precision of Neyman's model is marginal (Kaur et al., 2000). Kaur et al. (2000) proposed an optimum model for symmetric distributions which measure either only mid or extreme rank orders. They give the optimum model for both Mound shaped and U-shaped symmetric distributions. The Mound shaped distributions are those for which $\sigma^2_{(i:k)}$ is increasing in i for $1 \le i \le M$ and $\sigma^2_{(i:k)}$ is decreasing in i for $M \le i \le k$. U shaped distributions are those for which $\sigma^2_{(i:k)}$ is decreasing in i for $1 \le i \le M$ and $\sigma^2_{(i:k)}$ is increasing in i for $M \le i \le k$. Kaur et al. (2000) ignored rank orders with large variances and measured only rank orders having the smallest variances. To overcome this difficulty, Chandra et al. (2015) proposed new systematic allocation models for symmetric distributions.

3 NUMERICAL COMPARISONS OF RSS METHODS

3.1 Known Probability Distributions In the first case, it is assumed that the parent distribution from where the samples are drawn is known in advance. The performance of the proposed ARSS model with respect to SRS, balanced RSS and Neyman's optimum allocation model in terms of RPs was evaluated for some skewed and symmetric distributions. In skewed distributions, standard Lognormal [LN (0, 1)], Gamma [G(2), G(1), G(0.5)], Pareto [P(5), P(4.5), P(3)], Weibull [W(0.5)], Half Logistic and Half Normal distributions were used. For symmetric distributions, Standard Uniform, Standard Normal, Standard Special and Asymptotic Normal were used for the purpose of comparisons. The asymptotic normal distribution is the asymptotic approximation of a normal distribution. In skewed distributions, both, highly skewed [LN(0,1), G(0.5), P(3), P(4.5), P(5), W(0.5)] and moderately skewed [G(2), G(1)] are considered. The values of the mean and variances of order statistics for these distributions were taken from Harter and Balakrishnan (1996) and Sarhan and Greenberg (1962). For symmetric distributions, both Mound shaped [standard uniform] and U shaped [standard normal, standard special and asymptotic normal] distributions were considered to see the performance of different methods proposed in this paper. For the symmetric distributions, the values of the mean and variances of order statistics were obtained from Hastings et al. (1947). The values for C_k using Eq. (3) for each of the distributions were computed for $k = 2, 3, \ldots, 10$ and are presented in Table 1.

From Table 1, it is observed that, for even k, the values of C_k for skewed distributions decrease with increasing k. This means for infinitely large k; the correction factor will be close to zero and the proposed estimator $\overline{Y}_{(k)abal}$ is close to the original estimator of McIntyre (1952). For odd values of k, the values of C_k are negative and tends to zero, in general. For symmetric distributions, C_k was zero for each odd value of k. As with skewed distributions, the values of C_k decreased with increasing k for even k. The values of the three RPs (RP_1, RP_2, and RP_3) for skewed and symmetric distributions are presented in Table 2 and Table 3, respectively.

From Table 2, it is observed that for skewed distributions, the performance of proposed model is better than the balanced RSS as well as the Neyman's optimum allocation model for even set sizes. As is well known, RP_1 and RP_3 increase with increasing k. In the case of the proposed ARSS, it is observed that for moderately skewed distributions, RP_2 increased as k increased. However, for highly skewed distributions, RP_2 initially decreased with increasing values of k and then increased. For odd set sizes of skewed distribution, the proposed model does not perform better.

For symmetric distributions (Table 3), ARSS performed better with the odd set sizes for the mound shaped symmetric distributions than the other procedures; however, for even set sizes the RP is equal to the balanced RSS scheme. For U shaped distributions, the proposed method does not work. Therefore, based on these results, the proposed model may be considered as a good model for even set sizes of skewed distributions and odd set sizes of mound shaped symmetric distributions. Smaller set sizes are recommended when using the proposed ARSS method.

3.2 Unknown Distributions In the situations when the distribution under study is not known in advance, it is not possible to obtain the values of C_k for estimating the population mean. In this section, we propose a method to estimate C_k for different values of k. The expression for $\overline{Y}_{(k)abal}$ can be written as

$$\overline{Y}_{(k)abal} = S + C_k$$

where:

$$S = (1/aM\ ) \sum\nolimits_{i=1}^{M} \sum\nolimits_{j=1}^{a} Y_{(2i-1:k)j}.$$

We also know that $\overline{Y}_{(k)abal}$ is unbiased if

$$C_k = \mu - \frac{1}{M} \sum_{i=1}^{M} \mu_{(2i-1:k)j}$$

When the distribution is completely unknown, the estimated value of C_k is:

$$\widehat{C}_k = T - \frac{1}{M} \sum_{i=1}^{M} \widehat{\mu}_{(2i-1:k)j}$$

where $\widehat{\mu}_{(2i-1:k)j}$ is the estimated value of $(2i-1)^{th}$ order statistic ($i = 1, 2, \ldots, k$)based on the ordered observations taken before the actual measurement. This is explained by one real example of a survey related to forestry, given in the next subsection.

3.2.1 Performance Comparison for Real Data A survey was carried out with the purpose of estimating the total biomass (tonnes per 0.1 hectare) of even-aged chir pine (*Pinus roxburghii*), deodar (*Cedrus deodara*) and kail (*Pinus wallichiana*) trees in a forest of the Western Himalayas of India. The Above Ground Biomass (AGB) on 208 selected trees from the above forest having different altitudinal ranges was measured.

Table 1: Values of the bias correction factor C_k (Eq. 3) with b=0 for some skewed and symmetric distributions for k =2(1)10.

Distribution[1]	μ	σ^2	***Set size (k)***								
			k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10
Skewed Distributions											
LN(0,1)	1.649	4.671	0.858	-0.198	0.616	-0.221	0.496	-0.218	0.422	-0.208	0.371
G(2)	2.000	2.000	0.750	-0.088	0.493	-0.094	0.374	-0.090	0.304	-0.084	0.258
G(1)	1.000	1.000	0.500	-0.083	0.333	-0.089	0.256	-0.085	0.210	-0.079	0.179
G(0.5)	0.500	0.500	0.318	-0.074	0.220	-0.079	0.172	-0.075	0.143	-0.070	*[2]
P(5)	1.250	0.104	0.139	0.140	0.097	-0.033	0.077	-0.032	0.065	*	*
P(4.5)	1.286	0.147	0.161	-0.035	0.113	-0.039	0.090	-0.038	0.076	*	*
P(3)	1.500	0.750	0.300	-0.075	0.218	-0.084	0.178	-0.084	0.153	*	*
W(0.5)	2.000	20.000	1.500	-0.472	1.139	-0.528	0.947	-0.520	0.822	-0.498	0.733
Half logistic	1.371	1.386	0.598	-0.095	0.386	-0.101	0.289	-0.097	0.232	-0.092	0.195
Half Normal	0.798	0.364	0.330	-0.033	0.211	-0.035	0.157	-0.033	0.126	-0.030	0.106
Symmetric Distributions											
N(0, 1)	0.000	1.000	0.564	0.000	0.366	0.000	0.276	0.000	0.223	0.000	0.188
U(0,1)	0.000	1.000	0.577	0.000	0.346	0.000	0.247	0.000	0.193	0.000	0.158
Standard Special	0.000	1.000	0.535	0.000	0.365	0.000	0.284	0.000	0.236	0.000	0.203
Asymptotic Normal	0.000	1.000	0.431	0.000	0.294	0.000	0.227	0.000	0.187	0.000	0.159

[1]LN = Lognormal; G = Gamma; P=Pareto; W=Weibull; N=Normal; U=Uniform

[2]Some means and variances of order statistics in the literature were not found, these entries are denoted by *

AGB is easy to measure in a non-destructive way either by the use of allometric equations or by using the well-known Smalian/Huber/Newton formulae etc. For calculations, 208 trees were considered as the population (Appendix A) and AGB was taken as the concomitant variable for the purpose of ranking. The population mean and variance of this population is determined as 0.95632 and 0.91274 (tonnes per 0.1 ha), respectively. The small set sizes k=4 and 8 were selected for the purpose and five cycles were made. The selected order statistics and their actual measured total biomass under ARSS is given in Table 4. Total biomass measurement is a difficult task since it contains the below ground biomass as well that along the branches.

The values of $\widehat{C}_k$, RP_1 , RP_2 (based on Table 4) and RP_3 for two set sizes k=4 and k=8 were computed. The values of $\widehat{C}_k$ for k=4 and k=5 were calculated as 0.3198 and -0.4697 respectively. The values of RP_1 , RP_2 and RP_3 were obtained for k=4 as 1.753, 3.755 and 2.317 respectively. However, the same respective values for k=8 were obtained as 2.687, 4.722 and 4.075. As expected, the RP of ARSS with respect to SRS exceeds those to the balanced and Neyman's allocation methods with respect to SRS.

4 Discussion and Conclusions

McIntyre's (1952) approach of RSS requires measuring each order statistic and, therefore, the size of the sample should be at least the same as the set size k (for m=1). In forestry and environmental inventories, the measurement of sampling units is difficult and incurs higher costs and resources due to the frequent field visits and costs therein, however, the ranking of the units is very easy and takes minimum field visits. The minimum possible sample size for actual measurements with maximum precision is the primary objective of any multiphase sample design (Iles, 2003, 2012) and can improve experimental and research study results as well. The ARSS procedure was useful for reducing the sample sizes and measuring only the alternate order statistics. The method of ARSS has its benefits for estimating the populations mean for even set sizes of mound shaped symmetric distributions. For these cases, in general, the relative precision increases as the set size increases, and the estimator based on ARSS is uniformly better than SRS and balanced RSS and is also better than RSS with Neyman's method. The proposed estimator is biased. For the known distribution, the bias correction term may be taken from Table 1. Therefore, once

Table 2: Performance of different relative precision estimates (RPs; Eqs. 4-–6) for some skewed distributions for $k = 2(1)10$.

Distribution[1]	*RPs*	*k=2*	*k=3*	*k=4*	*k=5*	*k=6*	*k=7*	*k=8*	*k=9*	*k=10*
	RP_1	1.187	1.339	1.471	1.589	1.697	1.797	1.891	1.980	2.065
LN(0,1)	RP_2	8.693	0.977	7.155	1.073	7.046	1.175	7.177	1.273	7.376
	RP_3	1.578	2.120	2.639	3.141	3.630	4.109	4.580	5.044	5.502
	RP_1	1.391	1.753	2.096	2.759	2.742	3.052	3.354	3.650	3.940
G(2)	RP_2	2.909	1.529	3.613	2.043	4.360	2.538	5.090	3.018	5.798
	RP_3	1.502	1.987	2.460	3.444	3.387	3.842	4.294	4.743	5.189
	RP_1	1.333	1.636	1.920	2.190	2.449	2.700	2.944	3.181	3.414
G(1)	RP_2	4.000	1.359	4.114	1.747	4.619	2.125	5.174	2.492	5.731
	RP_3	1.528	2.039	2.538	3.029	3.512	3.991	4.464	4.934	5.401
	RP_1	1.245	1.483	1.696	1.905	2.071	2.277	2.512	2.632	*[2]
G(0.5)	RP_2	5.789	1.157	4.835	1.417	4.922	1.679	5.748	1.935	*
	RP_3	1.537	2.116	2.653	3.196	3.675	4.210	4.837	5.215	*
	RP_1	1.228	1.418	1.586	1.739	1.880	2.013	2.137	*	*
P(5)	RP_2	6.764	1.067	5.902	1.220	6.056	1.371	6.381	*	*
	RP_3	1.559	2.086	2.593	3.087	3.565	4.047	4.510	*	*
	RP_1	1.213	1.390	1.545	1.685	1.813	1.933	2.045	*	*
P(4.5)	RP_2	7.310	1.035	6.266	1.166	6.370	1.297	6.656	*	*
	RP_3	1.564	2.096	2.605	3.100	3.584	4.065	4.525	*	*
	RP_1	1.136	1.242	1.331	1.407	1.476	1.537	1.594	*	*
P(3)	RP_2	12.500	0.880	9.659	0.912	9.271	0.958	9.274	*	*
	RP_3	1.604	2.165	2.699	3.213	3.712	4.199	4.677	*	*
	RP_1	1.127	1.236	1.335	1.425	1.509	1.589	1.665	1.737	1.807
W(0.5)	RP_2	16.000	0.874	9.013	0.936	7.758	1.016	7.333	1.097	7.181
	RP_3	1.647	2.266	2.862	3.439	4.002	4.553	5.093	5.625	6.150
	RP_1	1.398	1.754	2.090	2.411	2.721	3.022	3.316	3.599	3.886
Half Logistic	RP_2	3.167	1.522	3.784	2.015	4.506	2.489	5.220	2.948	5.915
	RP_3	1.529	2.028	2.517	2.998	3.473	3.944	4.411	4.866	4.992
	RP_1	1.431	1.842	2.241	2.630	3.012	3.388	3.759	4.126	4.489
Half Normal	RP_2	2.509	1.715	3.284	2.398	4.106	3.062	4.921	3.715	5.722
	RP_3	0.571	0.571	0.571	0.571	0.571	0.571	4.394	4.865	5.334

[1]LN = Lognormal; G = Gamma; P=Pareto; W=Weibull; N=Normal; U=Uniform

[2]Some means and variances of order statistics in the literature were not found, these entries are denoted by *

the best possible probability distribution is fitted based on the sampled observations, ARSS methodology using the bias correction factor can be applied appropriately. In this methodology, those even set sizes of skewed and odd set sizes of Mound shaped symmetric distributions should be used which are nearer to the required sample size for the study. The advantages are seen in terms of gain in relative precision and its simplicity in real applications. The real-life example of a sample survey in forest also demonstrates the utility of the proposed ARSS. This methodology may also be useful in developing allometric equations for estimating the parameters like below ground biomass by providing a mechanism to select trees across the range of tree sizes based on ranks from smaller samples. The proposed method is strongly recommended for estimating the population means for even set sizes of skewed and odd set sizes of Mound shaped symmetric distributions and has immense utility in studies related to forestry and environment.

Acknowledgements

The authors declared no Conflict of interest. The authors gratefully acknowledge the efforts of Kim Iles, an anonymous reviewer and Editors for their efforts in improving the clarity of the manuscript.

Table 3: Performance of different relative precision estimates (RPs; Eqs. 4—6) for some symmetric distributions for k =2(1)10. (Note: U - Uniform distribution and N - Normal distribution)

Distributions	*RPs*	*k=2*	*k=3*	*k=4*	*k=5*	*k=6*	*k=7*	*k=8*	*k=9*	*k=10*
U(0, 1)	RP_1	1.500	2.000	2.500	3.000	3.500	4.000	4.500	4.500	5.500
	RP_2	1.500	2.222	2.500	3.316	3.500	4.364	4.500	5.392	5.500
	RP_3	1.500	2.010	2.526	3.046	3.569	4.095	4.623	5.152	5.682
N(0, 1)	RP_1	1.467	1.914	2.347	2.770	3.186	3.595	3.999	4.399	4.794
	RP_2	1.467	1.787	2.347	2.538	3.186	3.270	3.999	3.989	4.794
	RP_3	1.467	1.919	2.361	2.797	3.228	3.655	4.078	4.500	4.919
Standard Special	RP_1	1.401	1.752	2.072	2.371	2.654	2.924	3.183	3.434	3.745
	RP_2	1.401	1.459	2.072	1.874	2.654	2.258	3.183	2.619	3.745
	RP_3	1.401	1.793	2.177	2.552	2.922	3.287	3.648	4.006	4.488
Asymptotic Normal	RP_1	1.190	1.703	2.192	2.665	3.124	3.575	4.017	4.457	4.884
	RP_2	1.190	1.616	2.192	2.492	3.124	3.324	4.017	4.138	4.884
	RP_3	1.190	1.706	2.200	2.680	3.150	3.612	4.067	4.521	4.965

Table 4: Total biomass (tonnes per 0.1 hectare) of different order statistics for set size k=4 and 8.

Cycle no.	Set Size k=4		Set Size k=8			
	1	3	1	3	5	7
I	0.3006	0.4633	0.5282	0.6025	1.6855	3.9064
II	0.2036	0.4396	0.4598	0.6826	2.1625	2.8035
III	0.2455	0.5572	0.1730	0.5126	1.2586	2.8035
IV	0.2632	0.6186	0.3006	0.7162	0.8122	1.5508
V	0.7334	2.8035	0.4864	1.6400	2.1625	3.0108

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Appendix A

Details of trees measured for the above ground biomass and total biomass

Table A1. Deodar

Tree No,	Altitude (asl)	AGB (t/0.1ha)	total biomass (t/0.1ha)	Tree No,	Altitude (asl)	AGB (t/0.1ha)	total biomass (t/0.1ha)
27	1500-2000	0.92959	1.18988	127	2001-2500	0.20516	0.26261
28	1500-2000	1.50643	1.92823	128	2001-2500	0.44695	0.5721
61	2001-2500	2.32479	2.97574	129	2001-2500	2.1902	2.80346
62	2001-2500	0.38004	0.48645	130	2001-2500	0.3592	0.45977
63	2001-2500	0.2266	0.29004	131	2001-2500	0.5077	0.64986
64	2001-2500	0.77912	0.99728	132	2001-2500	0.2266	0.29004
65	2001-2500	0.71672	0.9174	133	2001-2500	0.53327	0.68258
66	2001-2500	0.30111	0.38542	134	2001-2500	0.8779	1.12371
67	2001-2500	0.60029	0.76837	135	2001-2500	1.01913	1.30449
68	2001-2500	0.64263	0.82257	136	2001-2500	0.60029	0.76837
69	2001-2500	0.53327	0.68258	137	2001-2500	0.57295	0.73337
70	2001-2500	0.53327	0.68258	138	2001-2500	0.31973	0.40926
71	2001-2500	0.33909	0.43404	139	2001-2500	2.35215	3.01076
72	2001-2500	0.43534	0.55724	140	2001-2500	0.81136	1.03854
73	2001-2500	0.60029	0.76837	141	2001-2500	0.82774	1.05951
74	2001-2500	0.3592	0.45977	142	2001-2500	0.31973	0.40926
75	2001-2500	0.49519	0.63385	143	2001-2500	0.27458	0.35146
76	2001-2500	0.29208	0.37386	144	2001-2500	0.60029	0.76837
77	2001-2500	0.27458	0.35146	145	2001-2500	0.27458	0.35146
78	2001-2500	0.89496	1.14555	146	2001-2500	0.26611	0.34062
79	2001-2500	0.24184	0.30955	147	2001-2500	0.48286	0.61806
80	2001-2500	0.47071	0.60251	148	2001-2500	0.57295	0.73337
81	2001-2500	2.72103	3.48292	149	2001-2500	0.5204	0.66611
82	2001-2500	0.21212	0.27151	150	2001-2500	3.17635	4.06573
83	2001-2500	0.55954	0.71621	151	2001-2500	0.30111	0.38542
84	2001-2500	0.5077	0.64986	152	2001-2500	4.03747	5.16796
85	2001-2500	0.48286	0.61806	153	2001-2500	0.34905	0.44679
86	2001-2500	0.98281	1.258	154	2001-2500	0.44695	0.5721
87	2001-2500	0.45874	0.58719	155	2501-3000	1.44042	1.84373
88	2001-2500	0.21926	0.28065	156	2501-3000	1.68944	2.16248
89	2001-2500	0.58653	0.75076	157	2501-3000	0.1097	0.14041
90	2001-2500	0.64263	0.82257	158	2501-3000	0.91219	1.16761
91	2001-2500	0.49519	0.63385	159	2501-3000	0.89496	1.14555
92	2001-2500	0.43534	0.55724	160	2501-3000	0.12395	0.15865
93	2001-2500	0.38004	0.48645	161	2501-3000	1.7847	2.28441
94	2001-2500	0.41267	0.52822	162	2501-3000	2.1902	2.80346
95	2001-2500	0.43534	0.55724	163	2501-3000	0.53327	0.68258
96	2001-2500	0.57295	0.73337	164	2501-3000	1.05613	1.35184
97	2001-2500	0.47071	0.60251	165	2501-3000	0.8779	1.12371
98	2001-2500	3.05188	3.90641	166	2501-3000	1.46226	1.8717
99	2001-2500	0.28324	0.36254	167	2501-3000	1.05613	1.35184
100	2001-2500	0.42392	0.54261	168	2501-3000	1.68944	2.16248

Table A1. Deodar (Continued)

Tree No,	**Altitude (asl)**	**AGB (t/0.1ha)**	**total biomass (t/0.1ha)**	**Tree No,**	**Altitude (asl)**	**AGB (t/0.1ha)**	**total biomass (t/0.1ha)**
101	2001-2500	0.38004	0.48645	169	2501-3000	0.92959	1.18988
102	2001-2500	0.47071	0.60251	170	2501-3000	1.05613	1.35184
103	2001-2500	0.17326	0.22177	171	2501-3000	0.74757	0.95689
104	2001-2500	0.21926	0.28065	172	2501-3000	2.46306	3.15271
105	2001-2500	0.30111	0.38542	173	2501-3000	2.86932	3.67273
106	2001-2500	0.5077	0.64986	174	2501-3000	0.38004	0.48645
107	2001-2500	0.25783	0.33003	175	2501-3000	1.25104	1.60133
108	2001-2500	0.54631	0.69928	176	2501-3000	1.13211	1.4491
109	2001-2500	0.5204	0.66611	177	2501-3000	0.71672	0.9174
110	2001-2500	0.57295	0.73337	178	2501-3000	1.19083	1.52427
111	2001-2500	0.33909	0.43404	180	2501-3000	2.1374	2.73588
112	2001-2500	0.39073	0.50014	181	2501-3000	2.1902	2.80346
113	2001-2500	0.48286	0.61806	182	2501-3000	0.60029	0.76837
114	2001-2500	0.45874	0.58719	183	2501-3000	5.52733	7.07498
115	2001-2500	0.47071	0.60251	184	2501-3000	0.57295	0.73337
116	2001-2500	0.36953	0.47299	185	2501-3000	0.21212	0.27151
117	2001-2500	0.31973	0.40926	186	2501-3000	0.19182	0.24554
118	2001-2500	0.29208	0.37386	187	2501-3000	0.60029	0.76837
119	2001-2500	0.45874	0.58719	188	2501-3000	0.82774	1.05951
120	2001-2500	0.70155	0.89799	189	2501-3000	0.11296	0.14459
121	2001-2500	0.2266	0.29004	190	2501-3000	1.17109	1.499
122	2001-2500	0.39073	0.50014	191	2501-3000	0.15642	0.20022
123	2001-2500	2.1902	2.80346	192	2501-3000	0.19182	0.24554
124	2001-2500	2.1902	2.80346	193	2501-3000	0.13225	0.16928
125	2001-2500	2.86932	3.67273	194	2501-3000	0.19182	0.24554
126	2001-2500	0.3592	0.45977				

Table A2. Kail

Tree No,	**Altitude (asl)**	**AGB (t/0.1ha)**	**total biomass (t/0.1ha)**	**Tree No,**	**Altitude (asl)**	**AGB (t/0.1ha)**	**total biomass (t/0.1ha)**
179	2501-3000	0.09024	0.11551	202	2501-3000	0.29075	0.37216
195	2501-3000	1.70322	2.18012	203	2501-3000	0.09729	0.12453
196	2501-3000	1.6553	2.11879	204	2501-3000	0.21495	0.27513
197	2501-3000	1.85071	2.36891	205	2501-3000	1.38121	1.76794
198	2501-3000	1.21153	1.55076	206	2501-3000	1.70322	2.18012
199	2501-3000	1.05262	1.34735	207	2501-3000	1.17079	1.49861
200	2501-3000	1.29504	1.65765	208	2501-3000	2.05595	2.63162
201	2501-3000	0.5034	0.64436				

Table A3. Chir pine

Tree No,	**Altitude (asl)**	**AGB (t/0.1ha)**	**total biomass (t/0.1ha)**	**Tree No,**	**Altitude (asl)**	**AGB (t/0.1ha)**	**total biomass (t/0.1ha)**
1	1500-2000	0.00403	0.34057	32	1500-2000	0.15907	0.20362
2	1500-2000	0.00508	0.70468	33	1500-2000	0.34341	0.43956
3	1500-2000	0.17176	0.21985	34	1500-2000	0.23486	0.30062
4	1500-2000	0.2056	1.63996	35	1500-2000	0.11851	0.15169
5	1500-2000	0.22736	1.68547	36	1500-2000	0.56217	0.71958
6	1500-2000	0.22736	0.39401	37	1500-2000	0.2824	0.36148
7	1500-2000	0.23486	0.29103	38	1500-2000	0.4725	0.60481
8	1500-2000	0.26607	0.30062	39	1500-2000	0.73781	0.9444
9	1500-2000	0.30782	0.29103	40	1500-2000	1.11076	1.42177
10	1500-2000	0.30782	0.39401	41	1500-2000	0.1917	0.24538
11	1500-2000	0.30782	1.55082	42	1500-2000	0.17176	0.21985
12	1500-2000	0.32537	0.67537	43	1500-2000	0.55053	0.70468
13	1500-2000	0.36193	0.56434	44	1500-2000	1.38934	1.77835
14	1500-2000	0.42042	0.81219	45	1500-2000	0.52763	0.67537
15	1500-2000	0.44089	0.53814	46	1500-2000	0.13516	0.17301
16	1500-2000	0.46184	0.00516	47	1500-2000	0.25809	0.33035
17	1500-2000	0.48329	0.59116	48	1500-2000	0.48329	0.61861
18	1500-2000	0.49419	0.61861	49	1500-2000	0.48329	0.61861
19	1500-2000	0.52763	0.39401	50	1500-2000	0.23486	0.30062
20	1500-2000	0.55053	0.41647	51	1500-2000	0.37137	0.47536
21	1500-2000	0.63452	0.63256	52	1500-2000	1.31677	1.68547
22	1500-2000	1.21157	0.26317	53	1500-2000	0.34341	0.43956
23	1500-2000	1.21157	0.00651	54	1500-2000	0.14688	0.188
24	1500-2000	1.28122	0.46327	55	1500-2000	0.29075	0.37217
25	1500-2000	1.31677	1.55082	56	1500-2000	0.45131	0.57767
26	1500-2000	0.48329	0.61861	57	1500-2000	1.28122	1.63996
29	1500-2000	0.23486	0.30062	58	1500-2000	0.29075	0.37217
30	1500-2000	0.40044	0.51256	59	1500-2000	0.48329	0.61861
31	1500-2000	0.57392	0.73462	60	1500-2000	0.50522	0.64668