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FORKS IN MY ROAD TOWARD MORE MATHEMATICS IN FOREST DYNAMICS MODELS

(AN INVITED AUTOBIOGRAPHICAL NOTE)

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Abstract. This note describes personal fits and starts in the authors career seeking an education to permit a serious use of applied mathematics in forest dynamics research. The personal journey through mathematics, while maintaining roots in forest dynamics, was not very efficient, and the sought after goal of universal description remains only partially tested. It would appear that significant improvement has been made in representing sugar maple height growth, based on application of 14 different differential equation models found in the literature over the last 175 years, but causal environmental variables are not identified.

Keywords: Forest dynamics models; differential/difference equations; applied mathematics; significant digits in parameter estimates; forks in career path.

Forward

Recall the great American baseball player, Yogi Berra, once remarked:

> "*If you come to a fork in the road, take it.*"

Surely, a socially responsible forester's first response would be:

> "*if you come to a fork in the road, pick it up, i.e., don't litter.*"

Here I would like to briefly recapitulate some '*forks in the road*' I've encountered in my attempts to do applied mathematics in forest dynamics research.

1 The Beginning

It all began at an early age. My first year in school my district initiated a kindergarten class, and the first day there were way too many 4 year olds for one teacher. She said – "Class we are going to have a test so that some of you can go straight to first grade. Who among you can tie your shoe laces? Raise your hands." My older sister had taught me, so I went to first grade, thereby ensuring that I was the youngest in the class (along with M.W.) for my entire schooling. I was a sophomore in high school, about 14, when I decided I wanted to be a forester. I had just turned 17 when I enrolled in forestry at Iowa State College. My senior year I took a series of statistical methods courses normally taken by forestry graduate students. Did quiet well. It wasn't that I was all that smart, but I do think I became wiser as I matured. Plus, I had a wonderful undergraduate mentor in Prof. George W. Thomson.

2 Graduation and Peace Corps

Upon graduation I had the opportunity to attend graduate school at University of Idaho. This year was a great boost for my 'mathematical' competence and confidence. University forestry advisers wanted all graduate students to take calculus – if you hadn't previously. I did and the instruction was so good, I took a linear algebra course the second semester. I got to know some fine faculty members at Idaho, as well as a great cadre of graduate students, but encountered another 'fork'. My interest was in continuous forest inventory, and they were doing that at Purdue, not at University of Idaho. So, I took that fork and headed for Lafayette. Finished up on time ('61), and took a fork to U.S. Peace Corps for two years: '61-'63 on St. Lucia, West Indies, averting by one day the fork that lead to Viet Nam. I think I was the first forester in U.S. Peace Corps.

3 Back to Iowa State

After Peace Corps I returned to Iowa State and enrolled in some courses I thought were right for post-Peace Corps me, but my heart was not into them. So, I quit Iowa State in the middle of finals week, Spring 1964, (taking an F in symbolic logic that is still on my transcript) and took an instructorship at SIU, Carbondale for '65. When that position ended I had by then regained my senses and went back to Iowa State to ask about resuming my studies – but with emphasis on applied mathematics. The forestry Dean was still extremely angry, because I had quit during finals week, and said he would have to submit my request for a faculty vote. In brief, I was allowed to enroll again as a forestry graduate student — with a Dean-insisted "NOT A CANDIDATE FOR A DEGREE" written in my Graduate School folder. No problem!

4 Diving into Math

I dove into mathematics with a vengeance: calculus II, differential equations, discrete math, symbolic logic, linear algebra, vector analysis, advanced calculus I & II. One year during 2 summer sessions I took 5 math courses — 3 the first session, 2 the second. I was doing quit well, because about that time, Prof. Ken Ware was leaving Iowa State to accept a position in Georgia, and someone asked him – 'want to take Leary with you to GA'? From what I heard, his reply was 'naaa'. Just as well, for a variety of reasons.

5 Back to Perdue

Otis Hall arranged for a University graduate fellowship for me to return to Purdue in 1966. There, much time was spent reading classical mathematical biology books by Volterra, Lotka, Gause, and other, mostly European, mathematical biologists. I also took courses in numerical analysis and boundary value problems in partial differential equations, because I had encountered the concept of a boundary value problem, and it seemed like it could be applied to response surfaces, standing crop amounts, and forest dynamics. (I would be remiss if I failed to mention the dissertations of Kenneth Turnbull and Leon Pienaar, and fellow Purdue graduate student John Moser. Turnbull had apparently been reading those classical mathematical biology writings much earlier, and probably the first 100 pages of his dissertation reflect this connection. Somehow he 'jumped ship' and adopted a regressionist's perspective and used the Richard's function for his research. Leon Pienaar followed suit.)

6 Dissertation

My final dissertation issue became: what is the partial differential equation governing stand dynamics – to which the boundary conditions can be applied to localize a solution. I didn't know enough mathematical ecology to postulate such an equation, so retreated to a simultaneous system of ordinary differential equations to govern dynamics of different tree size classes. I purposely kept the right-hand side sufficiently simple to be able to state the model components in plain English, something I appreciated in those classic books by Lotka, Gause, and Volterra.

$$\frac{dV}{dt} = aV \exp -bV$$

with rationale: 'that which results from biological growth is itself typically capable of growing $(aV)'$, and 'the flux of moisture, nutrients, heat and light into a fix physical space occupied by V, is limited, thereby limiting $V(\exp -bV)'$. The right hand side could not easily be integrated analytically to generate a closed form solution, but could be integrated both numerically and symbolically with Mathematica. For my purposes, iteration from initial conditions was sufficient. There had by then developed a group of system identification folks at Rand Corporation (Richard Bellman, Robert Kalaba, Harriet Kagwada) that were treating parameter estimation problems as nonlinear boundary value problems in ordinary differential/difference equations. (An early paper dealt with orbit determination of satellites as multipoint boundary value problems that could be solved using quasilinearization.) In my case, parameters a and b (above) could be estimated from CFI repeat measurements using system identification methods that used the multipoint boundary value method called quasilinearization.

7 USDA Forest Service

From Purdue the road forked to St. Paul, Minnesota and the US Forest Service, North Central Forest Experiment Station. I was fortunate to land a position on Allen Lundgren's production economics research work unit where I was to develop growth models to supply input to economic analyses of forest investments. The models were based on first order differential / difference equations, and since tree size has a great influence on value, we adopted a size class model where each size was governed by a differential equation that was coupled to quantities in larger tree sizes in right-hand-sides. Within about 1 year of being hired I was invited to participate in a "workshop for research on growth of mixed hardwood stands" held at Athens, GA, 3/10-11/1970, where I spoke about "Mathematical characterization of mixed stand development", and outlined the parameter estimation tasks as solving a multi-point boundary value problem in ordinary differential/difference equations. Later in the program Lew Grosenbaugh delivered his paper where he stated, in essence, since the dependent variable we directly observe is size, not growth, differential equations have no role to play in growth models. Oh, really? Was Lew advocating subtracting two numbers nearly the same to form the dependent variable in regression analysis and later treating this difference as the observation to be subtracted again from the predicted difference, with the product to be minimized? Isn't that minimizing squared double differences? By this time Lew was grade GS-16 and leader of the first Pioneering research work unit in the US Forest Service Research. And I was a miserly green GS-12. Perhaps my first question of Lew should have been — which fork will get me out of Athens the fastest? So, in retrospect it wasn't for me 'close encounters of the third kind', but more like two near misses with UGA.

8 Significant Digits

In point of fact, many models were developed around the Richards function (also called Chapman Richards model, I think), wherein successive tree or stand measurements were differenced to derive estimates of dependent variable change, thereafter used as observations on the dependent variable. My Purdue course in numerical analysis warned against doing exactly that – subtracting two numbers nearly the same (not a problem) and using the differences so-determined in further computations (big problem) – probably the worst possible mistake in numerical analysis practice. Significant digits lost from instrument readings (observations), can never be regained computationally. Further, the least count on, for example, dbh tapes, of that day, were unchanged at 1/10 inch, regardless of the tree diameter. [Wouldn't it seem logical to have narrower d-tape graduations for, say, the 3" to 10" zone of the diameter tape than for 30+ inch diameter zone?] So, the first rule of numerical measurement is "get significant digits during measurement by having appropriate least count for the measuring tool, and for sure, keep them during computation". If in doubt, learn Mathematica and use its feature that tracks precision (significant digits) through a long series of computations. Work done by Forest Inventory and Analysis research work units comes to mind, say as part of an 'audit' of their data processing procedures — probably already been done and filed in their QA/QC program documents.

9 Mathematical Ecology

My experience in Athens in 1970 hinted that perhaps mathematical ecologists, rather than forest biometricians, would provide a more accepting audience for differential/difference governing equations with the boundary value approach to parameter estimation. So I followed an example of compartment analysis done by Oak Ridge National Lab mathematical ecologist George Van Dyne, and would you believe that Ken Skog, my summer student programmer at the time, and I came up with different transfer coefficients than those published by Van Dyne. A short manuscript was prepared and sent to Ecology. 6 months passed and no peer review. 12 months passed; 18 months passed; 24 months passed before final review and acceptance. Publishers of Ecology even neatly omitted the date the manuscript was received in the final printing. I began to think, not in terms of forks on my career road, and more like one huge roundabout with no exit roads. Needless to mention, perhaps, were my fond reminiscences of hours on the tractor mowing hay, cultivating corn, etc., back on the Iowa farm.

10 Differential and Difference Equations

Why fixate on differential/difference equations, you might ask? Well, it all goes back to the fact that rhs of differential equations can be simple algebraically, but generate complex dynamics because of extreme sensitivity to initial conditions. Much of the explosion of interest in chaos analysis is based on that simple principle. Further, simple algebraic forms more easily allow giving numerical constants a physical, chemical, soil texture, etc, interpretation thereby changing a descriptive equation ("What is the character of") into a predictive equation (answer to a "What if" question), and perhaps even, an explanatory equation (answer to a "Why" question). After all, the advance of a scientist's career should be to move from singular descriptions toward predications and on toward universal explanations.

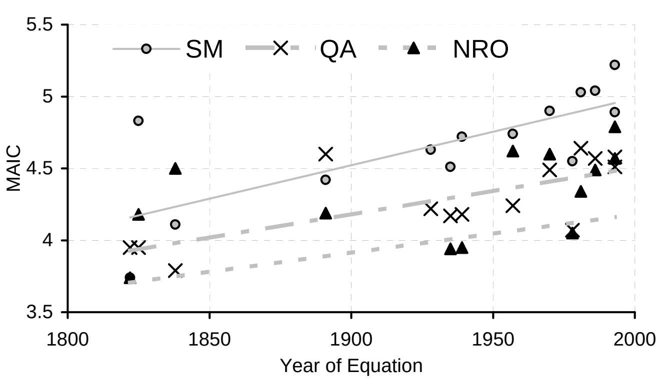

Figure 1: Modified Akaike information criterion (MAIC) for 14 models fit to sugar maple (SM), quacking aspen (QA), and northern red oak (NRO), height growth of trees growing in the Lake States, USA.

11 Equations, Equations

Figure 1 illustrates trends in performance of 14 different growth models (9 from a survey by Kviste and 2 modifications by Zeide) to estimate three tree species heights in Lake States, USA, showed a steady increase in 'goodness of fit' from 1822 (year Hossfeld IV was introduced) to 1993 (year Zeide proposed slight modification of Schnute's 2nd order differential equation). Figure 1 model identifiers, from left to right, are: Hossfeld IV, Gompertz, logistic (Verhultz), monomolecular, Yoshida I, Levakovic I, Korf, General Bertalanffy, Leary, Weibull (Yang), Schnute, Umemur/Hamlin, Schnute/Zeide, Leary/Zeide (Perhaps additional equations have been developed since 1993.) The model algebraic forms are in Leary and Johannsen (2010). Clearly the Gompertz was about 150 years ahead of its time, and the Weibull (W) was about 50 years behind its time. Equation algebraic forms are in Leary and Johannsen (2010).

To keep extending this relationship in future years, someone needs to focus on applied mathematical equations — second order differential and/or integro-differential equations.

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