

# COMPARING PROPERTIES OF SELF-REFERENCING MODELS BASED ON NONLINEAR-FIXED-EFFECTS VERSUS NONLINEAR-MIXED-EFFECTS MODELING APPROACHES

C.J. CIESZEWSKI <sup>1</sup>, M. STRUB <sup>2</sup>

<sup>1</sup>*University of Georgia, Athens, GA, USA*

<sup>2</sup>*Retired, Weyerhaeuser Co., Hot Springs, AR, USA*

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**ABSTRACT.** In this study, we compare the properties of self-referencing models, such as various site dependent growth and yield models for predictions of height, diameter, basal area, volume, and density, developed using Nonlinear-Fixed-Effects (NFE) versus Nonlinear-Mixed-Effects (NME) modeling approaches. The properties investigated include the following core traditional well-behaved model characteristics applicable to self-referencing functions: Base-Age-Invariance, Path-Invariance, Indifference Under Model Reparameterization, and Model Conditioning to have the predictions at the base-age equal to the reference point, as well as estimation and prediction statistics such as bias and variance of the fitted versus predicted residuals. The results of this investigation demonstrate that self-referencing models based on the NFE approach possess all the desirable properties associated with logical behavior of the model and estimation statistics, while the NME based self-referencing models lack the well-behaved model properties. We illustrate these properties using an example of fitting self-referencing models to panel data of loblolly pine age-height measurements on a range of sites from the South Africa Correlated Curve Trend Study.

**Keywords:** Mixed-Effects; Fixed-Effects; Self-Referencing; Base-Age-Invariance; Path-Invariance; Invariance Under Reparameterization; Well-Behaved Models; Model Conditioning; Site Models.

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## 1 BACKGROUND

### 1.1 Relevance of Self-Referencing Models and Types of Data

The self-referencing models are equations that use as input a point of reference defining the curve that passes through the point, as in the case of initial condition equations or boundary point solutions. Models based on such equations enable the users to use existing snapshot observations from a given population to simulate their dynamics more accurately than in the case of using broad population averages. The ability to use a known value of a snapshot observation in prediction models plays a vital role in the analysis of forest growth and yield dynamics in forest management, especially in monospecific even-age stands, such as tree plantations or fire origin regenerated stand populations. Such ability may be less critical but also helpful in other applications (e.g., see Cieszewski et al. 2013, Eq. 8).

The self-referencing functions are calibrated on repeated measures data, and are applicable to all essential stand characteristics such as height, diameter, volume, taper, and density; they are used for modeling all significant components of forest population dynamics including every aspect of growth and yield, mortality, and competition. The issues discussed in this article apply to all such models based on self-referencing functions, but for the clarity of the argumentation, we conduct this discussion using an example model from just one of the listed categories.

The height of dominant trees is an excellent example of an application of the self-referencing modeling, because tree height is a subject to functional changes across both the cross-sectional and longitudinal dimensions, while at the same time, the dominant or top height is the most stable stand characteristic that is less affected by crowding than other stand characteristics (Raulier et al. 2003) such as diameter, tree volume, taper, or mortality. The self-referencing site-height-age

models, hereafter called site models, are an essential component of the majority of growth and yield models in forestry and play an important role in forest management decision making and planning. Nearly all contemporary height growth models are based on self-referencing functions using an available observation of a population of interest, such as an inventory measurement, to define the growth trajectory corresponding to the subpopulation of interest.

Site models are typically developed either from stem analysis data or permanent sample plot data. Stem analyses usually provide longer time series records and allow for more effective analysis of changes over time. On the other hand, some practitioners argue that the permanent sample plot data are more realistic because dominant trees may change their positions over time either due to environmental conditions or due to physical damage (see Burkhart and Tomé 2012, section 7.3.3, for a broader review of the salient literature on the subject).

Regardless of which data are used for the self-referencing models they need to represent multiple series corresponding to different development potentials (e.g., site quality) with individual levels of performance. Since the different potentials are commonly unobservable variables, in the sense that they cannot be measured or observed explicitly, they are represented implicitly by a snapshot observation of the performance potential of any given series. The challenge in developing and fitting self-referencing models to such data is making the model capture unique characteristics of individual longitudinal series while accounting for common characteristics of the entire population. In practice, this can be accomplished through subject-specific fitting of all the series simultaneously to a common model, in which some of the parameters vary between different individuals, subjects or series.

Historically, there were various other approaches attempting to solve this problem, most of which were developed for site-dependent-height-age models, and they were evolving over time throughout the world. The simplest way to develop a site-dependent height-age model was fitting all the data to a mean curve, called the guide curve, and then adjusting the resulting curve by scaling it to pass through the reference point during the model application. This method was not modeling the characteristics of individual series and was eventually abandoned in favor of predominantly the fixed-base-age approach. The latter approach used one data point per series, which was frequently interpolated, as a site quality indicator, in a similar manner as it is done in the model implementation, treating this data point as the independent variable during the fitting of multiple curves. This method failed to produce Base-Age-Invariant parameter estimates ( $\text{BAI}_{\text{Par}}$ ), resulting in inconsistent pa-

rameters for different choices of base-age, and consequently, it has been largely abandoned. A more effective subject-specific approach was eventually adopted by using the varying-parameter methods, such as the Nonlinear-Fixed-Effects (NFE) modeling approach and subsequently the Nonlinear-Mixed-Effects (NME) modeling approach. Accordingly, most of the contemporary self-referencing models are developed using one of these two subject-specific approaches, and the purpose of this study is to investigate the properties of their resulting models and their predictions.

## 1.2 Self-Referencing Model Forms and Their Properties

The main properties of the self-referencing models, whether mathematical equations or algorithmic heuristics, are the Base-Age-Invariant model predictions ( $\text{BAI}_{\text{Pred}}$ ), Model Conditioning (MC) to predict values at a base age equal to the input values of the reference points, and Path-Invariance (PI). The PI property is similar to  $\text{BAI}_{\text{Pred}}$ , but it has broader applications including those in ageless models and difference equations. It means that the trajectory of height predictions over time in iterations is unaffected by different selections of steps executed for the predictions, such as in 1-year versus 5-year iterations. This also means that the projection of height from an initial age to middle age and then to a final age results in the same final height prediction as projecting the height at the final age directly from the height at the initial or any other age. This property results in consistent height predictions regardless of the sequence in which they are simulated. Clutter et al. (1983) point out the desirability of site index curves that pass through the site index at base age, which is MC. All the above three properties essentially mean that the self-referencing model generates unequivocally identical curves using any point on the curve, and they constitute the algebraic properties of the model formulation rather than the statistical properties of the model parameter estimates as in the case of  $\text{BAI}_{\text{Par}}$ .

Self-referencing models can be based on different mathematical equations. Early formulations of equations used for developing site models were incorporating a fixed base age site index (e.g.,  $H = f(A, S)$ ; where  $S$  is site index), defined as a height at a fixed arbitrary base age — usually 25, 50, 75 or 100 years. In practice,  $S$  in such models is not always consistent with the definition, which may cause problems with the model implementation.

Most contemporary mathematical formulations of self-referencing models are based on either the Algebraic Difference Approach (ADA) of Bailey and Clut-

ter (1974) or the Generalized Algebraic Difference Approach (GADA) of Cieszewski and Bailey (2000), who formalized the Cieszewski (1994) methodology. ADA and GADA dynamic site equations have desirable model properties including  $\text{BAI}_{\text{Pred}}$ , PI, and MC, with curves that go through  $S$  at base age. A substantially higher algebraic complexity of GADA models, which are generally more complicated mathematically than the fixed base age site models, has been somewhat a deterrent to their implementation. Nonetheless, there are many developments using such models (e.g., Adame et al. 2006, Cieszewski 2001–2003, (De los Santos-Posadas et al. 2006, Diéguez-Aranda et al. 2006a,b, Elfving and Kiviste 1997, Eriksson et al. 1997, Krumland and Eng 2005, Nord-Larsen 2006, Rivas et al. 2004, Trincado et al. 2003). On the other hand, many site models developed through other approaches, such as fixed base age site index, may not share the above properties.

### 1.3 Self-Referencing Model Parameter Estimation Properties

The main properties of parameter estimation are  $\text{BAI}_{\text{Par}}$  and Invariance Under Reparameterization (IUR). These are both statistical properties, which essentially mean that the estimated parameter values are unaffected by different selection of base age or by different parameterizations of the model forms used in model fitting.

Parameter estimation of site models has been evolving as much as the mathematical formulations. The oldest statistical method was based on the guide curve approach, which was merely fitting a mean height-age curve, subsequently scaled down or up to predicts low and high sites. Later subject-specific estimation methods were used to estimate the parameters of site models by treating  $S$  as observable variable taken directly from the data or data interpolations. The problem with this approach of site model parameter estimation is that the estimates of the site model parameters vary with different selections of base age. Since there is no right choice of base age for parameter estimation, this anomaly is a pitfall of base age-specific parameter estimation, which resulted in the eventual abandonment of this approach.

Bailey and Clutter (1974) introduced the application of covariance analysis for  $\text{BAI}_{\text{Par}}$  subject-specific parameter estimation. DuPlat and TraHa (1986) generalized this approach to nonlinear regression analysis using dummy variables, which is the Nonlinear Fixed-Effects (NFE) parameter estimation. Tait et al. (1988) and Cieszewski et al. (2000) show alternative programming approaches for implementation of the NFE subject-specific parameter estimation, which is not readily available in most commonly used software. The main ad-

vantages of NFE parameter estimation are the desirable  $\text{BAI}_{\text{Par}}$  and IUR.

Biging (1985), Lappi and Bailey (1988), and Lappi and Malinen (1994) have suggested using random parameter models for site model parameter estimation. Then, Lindstrom and Bates (1990) introduced the NME modeling approach, in which some of the parameters are treated as having random effects assumed to follow a random distribution (typically normal). Subsequently, Fang and Bailey (2001) and Calegario et al. (2005) among others advocated the use of the NME models for various types of forest growth and yield modeling.

NFE and NME approaches are both varying parameter methods, in which some parameters are common for all data, and some are subject-specific and vary between different series and are called hereafter site effects. The principal difference between the two approaches is that in the NME approach the site effects are assumed to be random, or have random component, and therefore, have to meet their distributional assumptions (e.g., normal distribution). In the NFE approach the site effects have no restrictions and can assume any values needed for the best fit, which means that, by definition, the NFE models must fit any data as good or better than a similar NME model. The same number of parameters can be made subject-specific in either of these approaches. Otherwise, the statistical models for both these subject-specific approaches are in general very similar.

Finally, it should be noted that just about any type of equation can be fitted with just about any type of the fitting methods discussed earlier. This topic, however, is not a part of our study. We do not discuss here such matters as, for example, fitting GADA models with guide curve or fixed base-age techniques, nor do we intend to discuss in detail various methods of model derivation or selection, which could be done with mathematics or fitting techniques, such as the NFE or NME modeling approaches. This study is concerned strictly with the comparison of fitting and predicting specific self-referencing functions with the NME versus NFE approaches, which are principally similar to each other but may vary in some of the well-behaved model properties discussed in this article.

### 1.4 Self-Referencing Model Prediction Properties

The main properties of well-behaved self-referencing model predictions are in part the same as for model forms and algorithmic heuristics, and they are the  $\text{BAI}_{\text{Pred}}$ , PI, and MC. Also, when considering model prediction, we look closer at the way the reference point is provided to the model and at the way the model is commonly used in its implementation, such as how  $S$  is

estimated, or what procedures are used for generating the predictions, as in the NME models.

While it is conceivable that someone may occasionally use a model for a tree or stand at an age equal to the base age, such an event is generally unlikely. Thus, predictions from fixed base age site models are generally obtained by first estimating or computing  $S$  values from data available at different ages. Estimated  $S$  values are then used as the input for the model prediction process to compute the height at the desired prediction age. If the predictions are part of an iterative simulation then the computed height is used to estimate  $S$  repeatedly through iterations (depending on whether the model predicts height at base age equal to  $S$  and how compatible the estimation of  $S$  is with height prediction) may or may not be  $\text{BAI}_{\text{Pred}}$ , PI, and MC. Thus fixed base age site models vary in their behavior and properties. In particular, models that don't have mathematically tractable solutions for  $S$  are prone to misuse and ill-conditioned predictions. Rose et al. (2003) demonstrated that an MC and  $\text{BAI}_{\text{Pred}}$  model (Cieszewski and Bella 1989), even when it's based on foreign data, predicts more accurate heights than a native, not MC and not  $\text{BAI}_{\text{Pred}}$ , model that was fitted to the local data.

Predictions of ADA and GADA based models are computed directly from height-age inputs. Those models are by design conditioned to meet the desirable and well-behaved model properties of  $\text{BAI}_{\text{Pred}}$ , PI, and MC.

In this study, we consider predictions from NME models that can be generated through the use of Best Linear Unbiased Predictors (BLUPS) or Empirical Best Linear Unbiased Predictors (EBLUPS). Since the NME modeling has extensive software support in SAS, making it easy to use even by inexperienced modelers, and EBLUPS produce small variances and biases, the NME modeling approach has been rapidly gaining popularity among forest practitioners. Some developments utilizing the NME modeling approach include Dorado et al. (2006), Sharma and Parton (2007), Meng and Huang (2009), Calegario et al. (2005), Fang and Bailey (2001), Wang et al. (2007), Adame et al. (2008), Calama and Montero (2004), and Saunders and Wagner (2008).

The literature lacks information about the properties of NME model predictions and their suitability for forest management practices and operational use. The general soundness of NME model EBLUPS predictions has not been investigated in detail in the forest modeling community. The purpose of this study is to investigate this subject and report the performance of the two subject-specific approaches according to historically established criteria for well-behaved model core properties.

## 1.5 The Objective of This Study

Our specific objectives were to use well-behaved self-referencing equations, with mathematical properties of MC and  $\text{BAI}_{\text{Pred}}$  of the fitted equation, to test the statistical properties of NFE and NME models with respect to their:

1.  $\text{BAI}_{\text{Par}}$  of model fitting,  $\text{BAI}_{\text{Pred}}$  of model predictions, and IUR;
2. MC to predict height at the base age equal to  $S$ ;
3. PI of the model predictions in iterative simulations;
4. The existence of any systematic deviations from the expected outcomes; and
5. Logical model behavior.

## 2 DATA

We used data from 15 plots in the South Africa Correlated Curve Trend (CCT) study to illustrate the properties of the two types of models resulting from NFE and NME modeling approaches. The data consists of measurements replicated at three locations and were originally described in Strub and Bredenkamp (1985); and therefore, they are not discussed here in detail. We used plots thinned to 300, 200, 150, 100 and 50 trees per acre before intra-specific competition for this study. Due to the low densities of the plots, the data (Figures 1, 2 and 3) do not show any apparent trends resulting from the differences in the stocking.

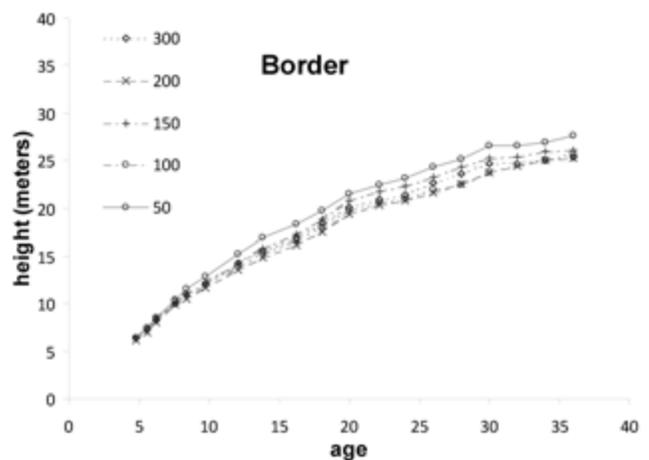


Figure 1: CCT data from Border, South Africa.

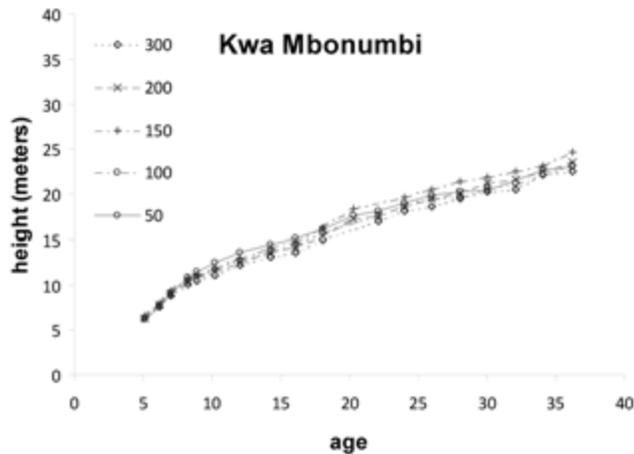


Figure 2: CCT data from Kwa Mbonumbi, South Africa.

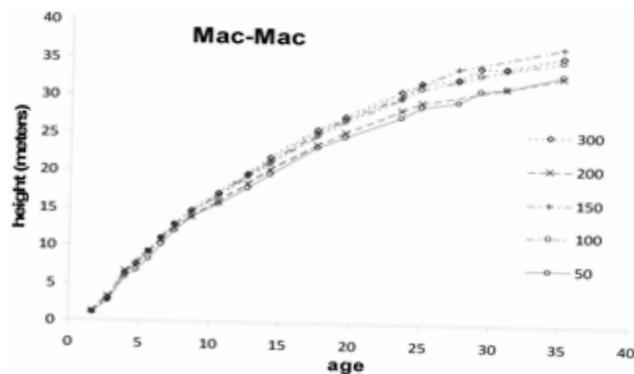


Figure 3: CCT data from Mac-Mac, South Africa.

### 3 METHODS

#### 3.1 Base Model Selection and Verification

To illustrate the differences between the NFE and NME parameter estimation and predictions, we selected the Schumacher (1939) function for the derivation of site equations:

$$H = e^{\alpha - \frac{\beta}{A}} \quad (1)$$

where:

- H is stand height;
- A is stand age,
- $\alpha$  and  $\beta$  are model parameters; and
- e is the base of natural logarithms.

Cieszewski and Bailey (2000) discuss derivations of several GADA models based on this function that are derived by modeling different relationships between  $\alpha$  and  $\beta$  across the range of site qualities. One of the simplest examples of their models is based on the linear relationship between the function parameters, which can be

accomplished by modeling each of them as proportional to an unobservable variable  $X$  (e.g.,  $\alpha = X$ ;  $\beta = \gamma X$ ) (see Cieszewski and Bailey 2000 Eq. 14). This is the basis for our comparative analysis of differences in model behavior in NFE versus NME parameter estimations and NFE versus NME model prediction approaches.

Following this example we reparameterized Cieszewski and Bailey (2000, Eq. 14) to a two-parameter site-height-age base model:

$$H = e^{X - \frac{\gamma X}{A}} \quad (2)$$

where:

- $X$  is an arbitrary unobservable variable of unknown magnitude that varies with site quality, or plot, or growth series, and which needs to be estimated by using empirical data; and
- $\gamma$  is the new global parameter that is common to all growth series and is estimated simultaneously with the estimation of  $X$  using empirical data.

Cieszewski and Bailey (2000, Eq. 15) recommend further reparameterization of the GADA formulation using the expected value parameterization originally proposed by Schnute (1981) and Ratkowsky (1990) which improves parameter estimation properties. We further modified the model to its implicit form using data-related reference points instead of the explicit unobservable variable  $X$ . The expected value parameterization of Equation (2) requires solving for  $X$  ( $X = f(H, A)$ ) and then substituting the solution with specific value  $X_0 = u(H_0, A_0)$  for  $X$  in Equation (2), which results in the following implicit dynamic equation:

$$H = e^{\ln(H_0) \left(\frac{A_0}{A}\right) \left(\frac{A-\gamma}{A_0-\gamma}\right)} \quad (3)$$

where:

- $A_0$  is an arbitrary reference age;
- $H_0$  is an arbitrary reference height, such as  $S$ ; and
- all other symbols are as previously defined.

#### 3.2 The NFE Site Models

To use more traditional terminology, we denote  $H_0$  as  $S$  and the base age as  $A_0$ . Then,  $\ln(S)$  is the natural logarithm of  $S$ , and only  $S$  varies by growth series. This formulation is commonly known as a fixed-base-age site model, except that in our formulation the base age is a variable and can be changed without affecting the equation properties as long as  $S$  corresponds to a selected base age. We will use both formulations of explicit Equation (2) and implicit  $S$  Equation (3).

The distinct characteristic of the NFE models is that only some of the model parameters are common for all

Table 1: Global parameter estimates and residual of sum of square errors (SSE) for NFE and NME models.

A0	NFE Models				NME Models			
	Eq. 4	Eq. 5			Eq. 6	Eq. 7		
	N/A	5	25	50	N/A	5	25	50
$\gamma$	2.99208	2.99208	2.99208	2.99208	2.99184	2.99191	2.99181	2.99179
SSE	163.4152	163.4152	163.4152	163.4152	163.4438	163.4435	163.4449	163.4452

series while the others, such as site effects, vary between the series. In statistical notation the explicit site-height-age Equation (2) fitted as a NFE model can be expressed as:

$$H_{ij} = e^{X_i - \frac{\gamma X_i}{A_{ij}}} + \varepsilon_{ij} \tag{4}$$

where:

- $i$  denotes individual growth series,
- $j$  is an individual age within a growth series; and
- $\varepsilon_{ij}$  is an independent normally distributed error term with mean zero and variance  $\sigma^2$ .

The fitting form of the implicit dynamic site-height-age Equation (3) becomes:

$$H_{ij} = e^{\ln(S_i) \left(\frac{A_0}{A_{ij}}\right) \left(\frac{A_{ij}-\gamma}{A_0-\gamma}\right)} + \varepsilon_{ij} \tag{5}$$

where  $S_i$  is the site index at base age  $A_0$  and is estimated from the data. All other notation is as previously defined.

### 3.3 The NME Site Models

What distinguishes the NME models from the NFE models is that they have an additional constraint that the varying parameter, or site effects  $X_i$  consists of a sum of its mean value and individual-specific deviations  $U_i$ , which are called random effects and must be randomly distributed. This, in addition to the estimation of the global model parameters, requires estimation of the mean and variance parameters for the assumed, typically normal, distribution, potentially additional covariance parameters for multiple random effects, and imposes constraints on what values the site effects can assume so that they conform to the assumption of the randomly distributed site effects.

The NME models are derived by substituting for the NFE,  $X_i$  that vary by growth series, or  $S_i$ , with a random parameter  $X + U_i$  or  $S + U_i$ .  $X$  and  $S$  are mean values for all growth series, and  $U_i$  is a random normal deviant with mean zero and variance  $\sigma_U^2$ . Accordingly, the explicit Equation (2) fitted as the NME model can be statistically defined as:

$$H_{ij} = e^{X+U_i - \frac{\gamma(X+U_i)}{A_{ij}}} + \varepsilon_{ij} \tag{6}$$

and the implicit dynamic site-height-age Equation (3) fitted using NME modeling can be written as:

$$H_{ij} = e^{\ln(S+U_i) \left(\frac{A_0}{A_{ij}}\right) \left(\frac{A_{ij}-\gamma}{A_0-\gamma}\right)} + \varepsilon_{ij} \tag{7}$$

where all the symbols are as previously defined.

### 3.4 Model Parameter Estimation and Model Predictions

We fit both of the NFE models (4) and (5) with the SAS NLIN procedure (version 9.3). We tested the fitting of the NFE Equation (4) directly, estimating  $X_i$  individually for each growth series. We also tested the fitting the NFE Model (5), estimating  $S_i$  individually for each growth series and each of base ages: 5, 25 and 50 years. The NFE model parameter estimation is straightforward and can be computed with the SAS NLIN procedure (version 9.3) or Excel using the Solver Add-in.

Using the NFE site Equation (5) we computed predictions for heights directly at ages 1 through 50 from the parameter estimates (Table 1) base ages 5, 25 and 50 and a range of  $S$ .

We fit both NME models (6) and (7) with the SAS NLMIXED procedure (version 9.3) using the assumption of normally distributed random effect.

After fitting the NFE and NME models, we examined the results of parameter estimation and sum of squared residuals. The primary objective of this examination, however, was to identify any parameter estimate discrepancies between estimates based on different base ages and model parameterizations, since it was evident that the NFE must always produce lower sum of squared residuals than the NME models.

Predictions consistent with the assumptions of the NME models are more complicated and involve optimization and numerical integration. We used the TECH=None option in the SAS NLMIXED procedure along with parameter estimates from the original dataset to estimate height at the same ages, base ages and range of  $S$  as in the case of the NFE model. The SAS NLMIXED procedure with TECH=None option is a better alternative than the BLUP or EBLUP method usually reported because it does not require a Taylor Series approximation used to derive the BLUP and EBLUP

estimates, but instead maximizes the likelihood numerically. Since  $S$  values that are not constrained to be equal to height at base age affect model behavior and its predictions we compared  $S$  values computed with the NME model with the conditioned  $S$  values of the NFE models.

Finally, since the use of growth models in multiple steps is more a standard than an exception we also tested the NME model predictions in multiple steps to illustrate the impact of lack of model conditioning for PI of model predictions.

## 4 RESULTS

### 4.1 Model Assumptions

The model of Cieszewski and Bailey (2000 Eq. 23) that was used (Eq. 2 and 3) for testing differences between the NFE and NME modeling approaches implies a linear relationship between the parameters of the Schumacher’s (1939) base Equation (1). Verification of this assumption was shown by fitting the base Equation (1) to all the individual growth series and analyzing the relationship between the parameter estimates. Examination of the parameter estimates from the fitting of Model (1) to individual growth series showed a reasonable linear relationship (Figure 4) between the parameters across the available range of site qualities for the CCT dataset. The fifteen pairs of parameter estimates conformed well to a linear relationship between the parameters with a good fit of the linear function with  $R^2$  of 0.839, which warranted our choice of the model for the analysis. The intercept was not significantly different from zero.

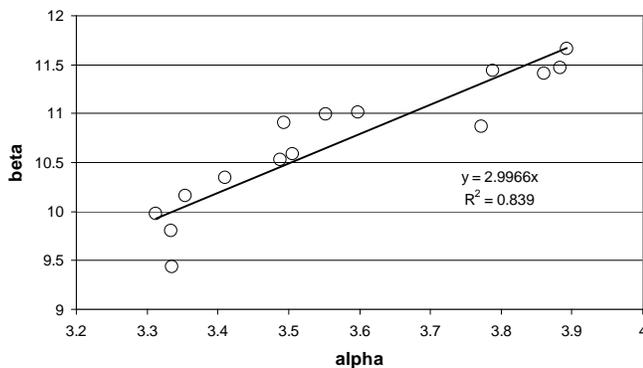


Figure 4: The relationship between parameter estimates of  $\alpha$  and  $\beta$  in the Schumacher (1939) model for the South Africa CCT data.

### 4.2 Model Fitting

As expected, fitting the implicit NFE Model (5) to all data using different base ages resulted in identical parameter estimates regardless of the different selections

of base age. To the contrary, the parameter estimates of the implicit NME Model (7) varied with a different selection of base age from 2.99179 to 2.99191 (Table 1), which demonstrates the lack of  $BAI_{Pred}$  in the NME modeling approach to parameter estimation.

Fitting the explicit formulation NFE Model (4) results in identical parameter estimation (Table 1) to estimates with the implicit formulation of the NFE Model (5) proving this NFE to be IUR. To the contrary, fitting the explicit formulation of the NME Model (7) results in different parameter estimates from fitting the implicit formulation of the NME Model (6), which demonstrates the lack of IUR of the NME modeling approach to parameter estimation.

As expected, the SSE was the smallest and identical in all the IUR NFE fitting of different model parameterizations. The SSE for the NME fitting is only slightly larger than the one resulting from the NFE fitting, which indicates that the normality assumption with regards to the random effects proved to be suitable for this data.

### 4.3 Model Predictions

All predictions of NFE models generated consistent curves regardless of the selections of base age (Figures 5, 6 and 7) and predicted the heights at the base age equal to  $S$ ; and therefore, the NFE model parameter estimation proved to be  $BAI_{Pred}$ , PI, and MC.

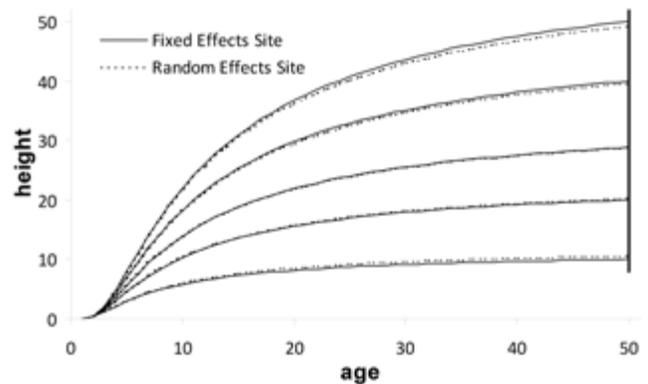


Figure 5: Fixed-Effects model curves (solid line) and Mixed-Effects model curves (dashed line) for base age 50 parameter estimates and site indexes 10, 20, 30, 40 and 50.

The NME model predictions generated using the TECH=None option in SAS NL MIXED procedure varied with different selections of base ages and site quality, for which the predictions were made, which demonstrate the lack of  $BAI_{Pred}$  in NME model predictions. The NME model predictions also failed to predict heights equal to  $S$  used to generate the predictions and deviated from the input reference point varying with both

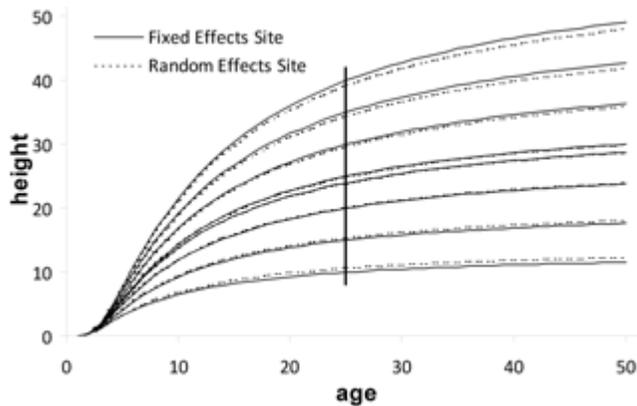


Figure 6: Fixed-Effects model curves (solid line) and Mixed-Effects model curves (dashed lines) for base age 25 and site indexes 10, 15, 20, 23.6, 30, 35 and 40.

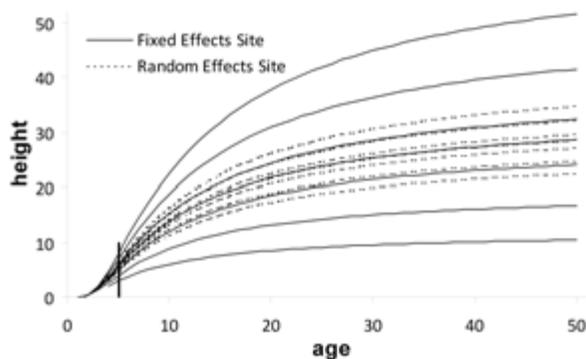


Figure 7: Fixed-Effects model curves (solid lines) and Mixed-Effects model curves (dashed lines) for base age 5 and site indexes 3, 3.5, 4, 4.2, 4.5, 5 and 5.5.

the base age and site quality; and therefore, it failed to be MC. Figures 5, 6 and 7 illustrate the results of generating NME model predictions for the assumed range of site qualities using the base ages 50, 25 and 5.

The smallest departure from expected prediction trends and from the height at the base age different from  $S$  was observed when the base age was high (Figure 5). For base age 50, the random curves were only slightly compressed toward the center mean curve. Only the global mean curve passes through  $S$  at base age; the other curves come very close to the height equal to  $S$  values at base age. In comparison, the NFE models pass through  $S$  at the base age in all scenarios.

As  $S$  base age used for generating the curves decreases the compression of curves towards the mean and departure from the height at base age equal to  $S$  grows larger. For base age 25 (Figure 6) the random curves were noticeably more compressed toward the center mean curve than for base age 50. Only the global mean curve passes through the height at base age equal  $S$ ; although the other curves still come quite close to  $S$  at base age. As

expected the NFE model passed through  $S$  at base age and generated the same curve as Figure 5.

For base age 5, the NME model curves were strongly compressed toward the center mean curve (Figure 7) and had large deviations at the base age from the input  $S$  values. Only the global mean curve passes through  $S$  at base age. As in all the other cases the NFE models pass at the base age through  $S$  values for all sites.

Using the model to simulate step predictions (Figure 8), as is frequently used in forest management practices, demonstrates that NME site model predictions fail to be PI in predictions for low and high sites; they are only PI at the global mean curve (the middle curve in Figure 8). The estimates for both high and low site quality data are biased towards the mean, which means that they are systematically underestimated for high sites and systematically overestimated for low sites, failing PI for predictions outside the global mean site.

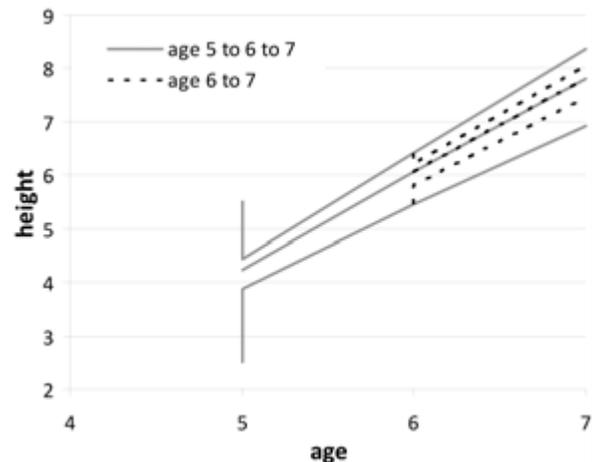


Figure 8: Base age 25 Mixed-Effect model  $S$  parameter estimates used to project from age 5 to age 6 and 7, and from age 6 to age 7.

## 5 DISCUSSION AND CONCLUSION

The results shown above are surprising since NME models have been studied extensively in Forest Biometrics research by theoretical statisticians and applied biometricians alike (e.g., Laird and Ware 1982, Leites and Robinson 2004, Meng and Huang 2009, Yum and Xu 2004, Hall and Bailey 2001). A number of scientists have also investigated the differences between the NFE and NME modeling approaches applied to various operational problems (e.g., Temesgen et al. 2008, Weiskittel et al. 2009), for comparing sampling designs (Segura-Correa et al. 2008), and predictions of sawmill dust exposure (Friesen et al. 2002). There have been some reports cautioning users of NME modeling about their limita-

tions and justifying development of sound model forms (Kershaw et al. 2009). Overton (1998) compares both approaches and concludes that they both have problems but that NME models can significantly overestimate variances when based on untenable assumptions. This is a peculiar finding because the assumptions about distributions of arbitrarily chosen nonlinear model parameters are approximations at best (e.g., a distribution of a value reaching from plus to minus infinity). Paulo et al. (2011) compared NFE and NME models for cork oak stands and noticed an overestimation of low diameter classes and underestimation of high diameter classes by the NME model. Still, the authors reported that overall the NME model performed better than the NFE model. Finally, Wang et al. (2008) failed to find any significant differences between the two types of model predictions applied to empirical data from pine after comparing both methods.

NME models are commonly advocated as robust and flexible. When compared against the NFE models the NME models are often claimed to be more flexible. Such claims are based on an ill founded assumption that the NME models can estimate more parameters as subject-specific. Indeed, some practitioners have fit NME site models specifying all of the model parameters as having random effects. Of course, the same can be done with respect to NFE models by defining all parameters as having subject specific fixed effects; although, such modeling just has not been found useful due to the lack of parsimony. Overall, the NFE modeling is by definition more flexible than the NME modeling approach since all the same functionalities such as different error structures (e.g., AR1, AR2, etc.) can be applied in either of the approaches, but the NME approach has more restrictions imposed regarding the distributional properties of the site effects.

We have compared the properties of the two contemporary approaches to self-referencing modeling in terms of the core well-behaved model criteria. To improve the quality of the predictions we used the TECH=None option in SAS NLMIXED procedure to estimate height at the same ages, base ages and range of  $S$  as in the case of the NFE model.

Our results reveal unexpected outcomes from the application of the NME modeling (not discussed in any earlier forestry literature on the subject). These outcomes throw a negative light on the interpretation and operational value of the NME modeling approach and confirm the well-behaved properties of the NFE modeling approach. In our analysis the NFE models were shown to have proper behavior regarding:  $\text{BAI}_{\text{Par}}$ , MC, and IUR in model fitting; and  $\text{BAI}_{\text{Pred}}$ , PI, lack of any systematic deviations in model predictions, and logical behavior.

To the contrary, testing the NME modeling approach demonstrated that the NME model lacked all the above properties. The NME modeling approach appears to be base age variant similar to fixed base age site model parameter estimates, varying with different selections of base ages used in the fitting process; the NME model parameter estimation resulted in inconsistent parameter estimates dependent on the model parameterization. This means that while the NME model estimates should depend only on the data and statistical assumptions, in case of the NME models it also depends on how the same model might be written or coded, which is undesirable since the parameterization of nonlinear models is strictly arbitrary and frequently subject to convenience, tradition, or coincident. The NME model predictions fail to be  $\text{BAI}_{\text{Pred}}$ , MC, PI, and IUR. In an environment of iterative simulations, the NME models produce different growth trajectories depending on a programmer or technician choice of the length of iteration steps. While the predictions of the NME models have overall low variance and small bias they systematically underestimate high sites and systematically overestimate low sites.

NME modeling is the state-of-the-art approach in the statistical theory, and it has many useful applications; however, self-referencing modeling is not one of them. From a theoretical point of view, NME models are not suitable for most forestry self-referencing modeling for various principal reasons in addition to lacking the well-behaved model properties. The data in forestry that are used for growth and yield self-referencing modeling are practically never random, but rather they are collected according to a model based design, typically with emphasis on uniform rather than random representation of sites and maximum practically achievable representation of old ages. The other reason is that it is typical in forestry modeling to use various model parameterizations of the same equations, which due to high nonlinearity can be sometimes reparameterized even between different model fittings just to change the model parameter characteristics or the search behaviour. Experienced modelers may explore such parameter transformations as for example  $p_1 \implies \ln(p_2) \implies e^{p_3} \implies p_4^2 \implies \frac{1}{p_5}$ , etc., to help with the model solutions to nonlinear searches or the estimates' characteristics.

In addition to  $\text{BAI}_{\text{Par}}$ , MC, and IUR, the NFE models as expected fit the data better with smaller SSEs. This is because the fitting process of the NME models imposes the additional constraint that the random effect  $U_i$  must follow an arbitrary assumption of randomness, which means that it must always have larger or equal sum of square errors when the same models are fitted to the same dataset as the NFE site models without the constraint. NFE self-referencing models ensure logical

behavior, well-behaved model properties, and the uniform unbiased representation of low and high sites.

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